Mirror symmetry and degeneration

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Mirror symmetry is a mysterious relationship, originally discovered by string theorists, between symplectic geometry and complex geometry.

Any symplectic manifold comes in a one-parameter family rescaling the symplectic form. The corresponding family of complex manifolds is typically a degeneration whose monodromy is maximally unipotent. Therefore, mirror symmetry is more about pairs of families of manifolds rather than pairs of manifolds, and various techniques to deal with degenerations, such as log geometry, rigid analytic geometry, tropical geometry, and Gromov–Hausdorff convergence, has been applied to mirror symmetry.

The prototypical example is a symplectic 2-torus with the unit symplectic form, which is mirror to the Tate curve. If we multiply the symplectic form by three, then the mirror becomes the Hesse family of cubic curves. These examples generalize to Calabi–Yau hypersurfaces in projective spaces.

When the symplectic form is exact, although multiplication of symplectic form does not change the symplectomorphism class of the symplectic manifold, one can have a degenerating mirror family by choosing a symplectic divisor. A typical example is the pair of a cylinder $S^1 \times \mathbb{R}$ and a point, which is mirror to the degeneration of the multiplicative group to the union of two affine lines. This generalize to a pair of a power of the cylinder and its divisor, which is mirror to a degeneration of a power of the multiplicative group.

In the talk, we will give a gentle introduction to mirror symmetry with an emphasis on the relation with degenerations.