Forcing and volume constraint in the crystalline mean curvature flow

Norbert Pozar

In this talk I will discuss new results on the crystalline mean curvature flow with given non-uniform forcing or a volume constraint. It is a model of the growth of crystals and it is formally a gradient flow of an anisotropic surface energy, whose minimizer for a fixed volume, the so-called Wulff shape, is a convex polytope. The surface of the evolving set develops corners and flat parts (edges and facets) that are usually preserved in the evolution, but they can also break and bend. The resulting velocity law $V = g(\nu, \kappa_\sigma + f(x,t))$ is nonlocal and the evolving surface can be expected to be only Lipschitz. We introduce a notion of viscosity solutions for this problem in an arbitrary dimension with nonuniform forcing $f = f(x,t)$, which we show is well-posed, and stable with respect to approximation by anisotropic mean curvature flow with smooth anisotropy. Our analysis is based on a variational understanding of the crystalline curvature κ_σ combined with the forcing f. We also consider the existence of solutions for the problem with a volume constraint, with initial data with a certain reflection property given by the symmetries of the Wulff shape, which we show is preserved in the evolution. We perform the analysis for the smooth anisotropic mean curvature flow and take advantage of the stability of solutions to extend the results to the crystalline case.

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