

Local Hecke algebras and newforms

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The theory of newforms was developed by Atkin-Lehner, Li-Miyake in the integral weight setting. The space of newforms is defined as the orthogonal complement to the space of old forms under the Petersson inner product. We give a direct characterization of the newforms space as a common eigenspace of certain finitely many pairs of conjugate operators. Our method is to use local Hecke algebras of GL_2 to get hold of these conjugate operators and relations amongst them. In the half-integral weight setting newform theory has been developed by Kohnen, Ueda and many others for certain levels but has been essentially restricted to the Kohnen plus subspace. We give a newform theory for the full space of half-integral weight forms of level $8M$, M odd and square-free and give an analogous characterization in terms of common eigenspace. In this case we describe certain genuine local Hecke algebras of the Kubota-Gelbart double cover of SL_2 by generators and relations and obtain local Shimura correspondence. This work is a part of an ongoing joint project with E. M. Baruch.