Topics in $\mathbb{A}^1$-homotopy theory

講演者：　Shane Kelly（東京工業大学）

（概要） One of the dominant themes in 20th century mathematics is homotopy theory, which studies topological spaces up to stretching and bending. Algebraic geometry on the other hand can be said to study polynomials via the spaces that they define. Due to the rigid nature of polynomials however, for a long time many homotopy theoretic techniques were unavailable in algebraic geometry. In the 90's, Morel, Voevodsky, and others developed a homotopy theory inside algebraic geometry that replaces the unit interval with the affine line $\mathbb{A}^1$. This allows a large selection of homotopy theoretic tools to be imported into algebraic geometry, and led for example, to a proof of Milnor's conjecture, which can be thought of as a classification of quadratic forms (over any field not of characteristic 2) using Galois groups.

This talk will begin with a very gentle introduction to some basic notions in $\mathbb{A}^1$-homotopy theory, and then move on to some more advanced topics such as: classification of algebraic vector bundles; quadratic forms and the Milnor Conjecture; motivic cohomology and Beilinson's conjectures on special values of L-functions; Ayoub's Conservativity Conjecture and Bloch's Conjecture; wild ramification and motives with modulus; the Grothendieck-Ayoub six functor formalism, applications to representation theory, and the Finkelberg-Mirković conjecture.