Discrete linear Weingarten surfaces

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In this talk, we will give a definition of discrete linear Weingarten surfaces in 3-dimensional space forms, in analogy to the case of smooth linear Weingarten surfaces in those space forms. Since the discrete surfaces have no canonical notion of differentiation, we must use alternate characterizations of smooth linear Weingarten surfaces to establish the analogy between the definitions in the smooth and discrete cases. For this purpose, an examination of the more general notion of Omega surfaces is useful. The naming "Omega surfaces" comes from Eisenhart's papers, or perhaps even earlier, and these surfaces exist in the setting of Lie sphere geometry, where isothermic sphere congruences are used to define them. With this approach in hand, one can characterize the linear Weingarten surfaces in 3-dimensional space forms as those Omega surfaces whose corresponding flat connections have constant conserved quantities. This viewpoint can then be readily adapted to define discrete linear Weingarten surfaces. In the special case of linear Weingarten surfaces of Bryant type, a discrete analog has already been established, and we will explain why the definition we give both matches and generalizes that previous definition.