CURVATURE ESTIMATES FOR SUBMANIFOLDS WITH PRESCRIBED GAUSS IMAGE AND MEAN CURVATURE

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ABSTRACT

We study that the $n-$graph defined by a smooth map $f : \Omega \subset \mathbb{R}^n \to \mathbb{R}^m, m \geq 2$, in $\mathbb{R}^{m+n}$ of the prescribed mean curvature and the Gauss image. Under the condition

$$\Delta_f = \left[\det\left(\delta_{ij} + \sum_\alpha \frac{\partial f^\alpha}{\partial x^i}\frac{\partial f^\alpha}{\partial x^j}\right)\right]^{\frac{1}{2}} < 2,$$

we derive the interior curvature estimates

$$\sup_{D_R(x)} |B|^2 \leq \frac{C}{R^2}$$

when $2 \leq n \leq 5$ with constant C depending on the given geometric data. If there is no dimension limitation we obtain

$$\sup_{D_R(x)} |B|^2 \leq CR^{-a} \sup_{D_{2R}(x)} (2 - \Delta_f)^{-\left(\frac{3}{2}+\frac{1}{s}\right)}, \qquad s = \min(m, n)$$

with $a < 1$. If the image under the Gauss map is contained in a geodesic ball of the radius $\frac{\sqrt{2}}{4}\pi$ in $\mathbf{G}_{n,m}$ we also derive corresponding estimates.

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