L^∞-Energy method and its applications

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1 Introduction

When one tries to investigate the existence, regularity and uniqueness of solutions of nonlinear partial differential equations, the suitable choice of the function spaces where one works is crucial for its success. Normally such nice function spaces are closely related with the various nature of the equations to be considered and it would be clear that there is no almighty function space for all equations.

However, the main purpose of this talk is to point out that as far as the uniqueness and local (in time) existence of solutions, L^∞-space could be the most suitable for a rather large class of nonlinear parabolic equations, including some quasilinear or strongly nonlinear parabolic equations, if we are equipped with "L^∞-energy method" developed in [1, 2].

This L^∞-energy method is a tool which makes it possible to derive energy estimates in L^∞ even when any energy estimates could not be expected in L^r with $1 \leq r < +\infty$, which is explained in the next section by using two simple but typical examples of nonlinear parabolic equations.

Roughly speaking, if the nonlinearity obstructing the energy estimates in L^r-spaces depends only on the solution u itself but not on the derivatives of u, we have only to establish the L^∞ a priori bound for u in proving the existence of the unique solution.

However, if that nonlinearity is composed of up to ∇u, we must deduce a priori estimates of u and ∇u in L^∞, such an example will be given.

Furthermore, our L^∞-energy method enables us to derive C^∞-estimates for a class of nonlinear parabolic equations including the porous medium equations.

It will be also exemplified that our method works well also for a strongly nonlinear parabolic equations governed by the so-called ∞-Laplacian which is not of the divergence form.

References

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