Compactifications of symmetric spaces and positive eigenfunctions of the Laplacean

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Let G be a semi-simple Lie group, with a finite center, K a maximal compact subgroup, $X = G/K$ the corresponding Riemannian symmetric space, L the associated Laplace-Beltrami operator. We denote by $-\lambda_0 > 0$, the upper bound of the $\mathbb{L}^2$-spectrum of L, by $G_\lambda(x,y) = (D - \lambda I)^{-1}$ the resolvent of $D = -L$. We are interested in the positive eigenfunctions of L and in the asymptotic behaviour of the Green kernel $G_\lambda(x,y)(\lambda \leq \lambda_0)$ when x is fixed and y tends to infinity in X.

We describe in geometrical terms the Martin compactification of X corresponding to the Martin kernel $K_\lambda(x,y) = \frac{G_\lambda(x,y)}{G_\lambda(0,y)}$ where 0 is the fixed point of K in X. The description involves the visual compactification of X and the Satake compactification of X. The proof make use of the asymptotics of Green kernels and spherical functions along the walls of the Weyl chambers in X.