Stationary solutions for the nonlinear Schrödinger equation with damping term

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We consider the following nonlinear Schrödinger equation with damping and external forcing terms:

$$\text{(LL)} \qquad \partial_t u - i\Delta u = -(1+i\theta)u + i|u|^2u + E_I, \qquad (t,x) \in \mathbb{R}^{1+n},$$

where $i = \sqrt{-1}$, $1 \le n \le 3$ and θ and E_I are positive constants. Here, u denotes the slowly varying envelope of the electric field, and θ and E_I denote the detuning parameter and the input field, respectively. Equation (LL) describes a physical model of a unidirectional ring or Fabry-Pérot cavity with plane mirrors containing a Kerr medium driven by a coherent plane-wave field (see Lugiato and Lefever, Phys. Rev. Lett., **58** (1989), 2209–2211).

We study the existence of stationary solution for (LL).

$$\text{(SLL)} \qquad -\Delta u + (-i\lambda + \theta)u - |u|^2u = -iE_I, \qquad x \in \mathbb{R}^n, \quad 1 \le n \le 3,$$

where λ is a small positive parameter. In the case $\lambda, E_I \ll \theta$, equation (LL) may be reduced to the cubic nonlinear Schrödinger equation from a physical point of view (see Firth and Lord, J. Mod. Opt., **43** (1996), 1071–1077). In this case, we have the following theorem concerning the existence of stationary solution for (SLL), which has recently been obtained in collaboration with Tomoyuki Miyaji and Isamu Ohnishi, Hiroshima University.

Theorem. *Let $1 \le n \le 3$ and $\theta > 0$. If λ and E_I are sufficiently small, then there exists a solution u in $H^2(\mathbb{R}^n)$ of (SLL) such that*

$$u = E_S + \phi + z(\lambda), \qquad z(\lambda) \to 0 \text{ in } H^2(\mathbb{R}^n) \ (\lambda \to 0),$$

where E_S is a solution of the algebraic equation $E_I^2 = -E_S - i(|E_S|^2 - \theta)E_S$ and ϕ is a positive radial solution in $H^2(\mathbb{R}^n)$ of the following equation.

$$-\Delta\phi + \theta\phi - 3|E_S|^2\phi - 3|E_S|\phi^2 - \phi^3 = 0, \qquad x \in \mathbb{R}^n.$$

Equation (SLL) has no variational structure because of the presence of damping term and so we employ the implicit function theorem around $(u,\lambda) = (E_S + \phi, 0)$.