One Forum, Two Cities 2013:

Aspect of Nonlinear PDEs

Abstract

September 17-20, 2013

Waseda University, Tokyo, Japan

One Forum, Two Cities 2013: Aspect of Nonlinear PDEs

September 17-20, 2013

Waseda University, Tokyo, Japan

Organizing Committee:

Chang-Shou Lin (National Taiwan University)
Mitsuharu Otani (Waseda University)
Eiji Yanagida (Tokyo Institute of Technology)

Local organizing committee:

Mitsuharu Otani (Waseda University)
Tohru Ozawa (Waseda University)
Yoshihiro Shibata (Waseda University)
Yoshio Yamada (Waseda University)

This workshop is supported by

Institute of Mathematical Fluid Dynamics, Waseda University
JSPS Grant-in-Aid for Challenging Exploratory Research (M. Otani)
No. 24654036
JSPS Grant-in-Aid for Scientific Research (A) (E. Yanagida)
No. 24244012

ABSTRACTS

ON UNIQUENESS OF BUBBLING SOLUTIONS OF CHERN-SIMONS-HIGGS EQUATION WITH DOUBLE VORTEX

Chang-Shou Lin

Abstract: We consider the following Chern-Simons-Higgs equation:

$$\Delta u + \frac{1}{\varepsilon} e^u (1 - e^u) = 4\pi N \delta_0 \text{ in } \Omega,$$

where Ω is the torus with a rectangle as its fundamental domain. We prove that for $N = 2$, ε small, the equation has exactly two solutions.

Traveling spots of the singular limit problems of FitzHugh-Nagumo equations

Hirokazu Ninomiya (Meiji University)

Abstract. In this talk, we consider traveling spots observed in two-dimensional excitable media. First, we introduce the singular limit problem of the FitzHugh-Nagumo equations. Then we show the existence of the traveling spot including the front and the back. This work is based on a joint work with Y.-Y. Chen and Y. Kohsaka.

Classification of the Entire Radial Self-Dual Solutions to Non-Abelian Chern-Simons Systems

Hsin-Yuan Huang
National Sun Yat-sen University, Taiwan, ROC
hyhuang@math.nsysu.edu.tw

In this talk, we consider the the entire radial self-dual solutions to non-Abelian Chern-Simons Systems

$$\begin{pmatrix}\Delta u\\ \Delta v\end{pmatrix} = -K\begin{pmatrix}e^u\\ e^v\end{pmatrix} + K\begin{pmatrix}e^u & 0\\ 0 & e^v\end{pmatrix}K\begin{pmatrix}e^u\\ e^v\end{pmatrix} + \begin{pmatrix}4\pi N_1\delta_0\\ 4\pi N_2\delta_0\end{pmatrix} \text{ in } \mathbb{R}^2, \quad (1)$$

where $N_i \geq 0$, $i = 1, 2$ and $K = (a_{ij})$ is a 2×2 matrix satisfying a_{11}, $a_{22} > 0$, a_{12}, $a_{21} < 0$ and $a_{11}a_{22} - a_{12}a_{21} > 0$. This system is motivated by the the relativistic non-Abelian Chern-Simons model, Lozano-Marqués-Moreno-Schaposnik model of bosonic sector of $\mathcal{N} = 2$ supersymmetric Chern-Simons-Higgs theory, and Gudnason model of $\mathcal{N} = 2$ supersymmetric Yang-Mills-Chern-Simons-Higgs theory. Understanding the structure of entire radial solutions is one of fundamental issues for the system of nonlinear equations. We prove that any entire radial solutions of (1) must be one of topological, non-topological and mixed type solutions, and completely classify the asymptotic behaviors at infinity of these solutions. As an application of this classification, we prove that the two components u and v have intersection at most finite times. This is joint work with Professor Chang-Shou Lin.

On some two phase problems with compressible and incompressible viscous fluid flow.

Yoshihiro Shibata (Waseda univ. Dept. Math. and RISE)

Abstract. In this talk, I will talk about some unique solvability of evolution of compressible and incompressible fluids separated by a closed interface without surface tension. The key step is to prove the existence of $\mathcal{R}$-bounded solution operator to the linearized problem. Then, we can drive the maximal L_p-L_q regularity and the generation of analytic semigroup for the linearized problem. Especially, in a bounded domain, we can show the exponential stability of our linearized problem. Thus, we can show a local in time existence of two phase problem for the compressible and incompressible viscous fluid flow in time L_p and space L_q framework under the Lagrangian coordinate. And also, we have a global in time existence theorem for some small initial velocity in a bounded container.

蔡岱朋 (Tai Peng Tsai)

Title:
Forward Discretely Self-Similar Solutions of the 3D Incompressible Navier-Stokes Equations

Abstract:
Extending the work of Jia and Sverak on self-similar solutions of the Navier-Stokes equations, we show the existence of large, forward, discretely self-similar (DSS) solutions for DSS initial data u_0 with factor λ, assuming either the DSS factor λ is sufficiently close to 1 according to the Holder norm of u_0, or if u_0 is axisymmetric with no swirl. I will also discuss their relevance to the uniqueness problem, and my joint work with D. Chae on the corresponding existence problem of DSS solutions for Euler equations.

Nonstationary flows of micropolar fluids with the spin-vortex interaction boundary conditions

Kei Matsuura (Waseda University)

Abstract. Micropolar fluid model is one of generalizations of the classical Navier-Stokes model. In this model the microstructure of the fluid is taken into account. The effect of microstructure is treated as the average field of the microrotation. In this talk, we will show some results concerning the incompressible microplolar fluid flows in smooth bounded domains with the spin-vortex interaction boundary conditions.

On a five-component Gierer-Meinhardt system with saturation

Kazuhiro Kurata (Tokyo Metropolitan University)

Abstract. Inspired by the recent work of J. Wei and M. Winter, we will consider a five-component Gierer-Meinhardt system with saturation, which has a long range activation effect. This system whose components are two activators, one inhibitor and two lateral activators, was proposed by Gierer-Meinhardt in 1980 to explain the phenomenon of mutual exclusion of two activators. Recently, Wei and Winter studied the stationary problem for this system in one dimensional case and proved the existence and stability of mutually exclusive spikes which are located in different positions. In this talk, as a first step to study this system in higher dimensional case, also including saturation effect, we present our recent results on several a prori estimates of stationary solutions to the five-component Gierer-Meinhardt system with saturation and its application to the study of the parameter range of non-existence of non-constant sataionary solutions. This is a joint work with Satoshi Takaku (Tokyo Metropolitan University).

吳昌鴻 (Chang- Hong Wu)

Title:
Spreading speed and traveling waves for a two-species weak competition system with free boundary

Abstract:
In this talk, we will discuss a Lotka-volterra type competition model with free boundary in one-dimensional habitat.
Based on the comparison principle for free boundary problems, we provide some estimates of the spreading
speed. Also, we deal with traveling wave solutions for the same model and show that there
exists a traveling wave solution. This talk is based on a joint work with Jong-Shenq Guo.

Bifurcation structure of stationary solutions of the Lotka-Volterra competition model with diffusion and advection

Kosuke Kuto (The University of Electro-Communications)

Abstract. This talk is concerned with the Neumann problem of a stationary Lotka-Volterra competition system with diffusion and advection. First we give sufficient conditions of the existence/nonexistence of nonconstant solutions. Next we derive a shadow system of the model as both diffusion and advection of one of the species tend to infinity. The shadow system can be reduced to a semilinear elliptic equation with nonlocal constraint. For the simplified 1D case, the bifurcation structure of nonconstant solutions of the shadow system can be classified according to the coefficients. For example, this structure involves a simple curve of nonconstant solutions which connects two different singularly perturbed states (boundary layer solutions and internal layer solutions). This talk is based on a joint work with Tohru Tsujikawa (University of Miyazaki, Japan).

Multiple points blowup for the two-dimensional Keller-Segel system

Yukihiro Seki (Tokyo Institute of Technology)

Abstract. We discuss blow-up of solutions to the parabolic-elliptic Keller-Segel system. It is well-known that if the total mass of cells is initially greater than 8π, then the corresponding solution blows up in finite time and has at most a finite number of blow-up points. In particular, the blow-up of any radial solution takes places only at the origin. On the other hand, it has not yet been proven whether or not solutions may actually have multiple blow-up points. In this talk I will show that, given any positive integer n, we may construct (non-radial) solutions with exactly n blow-up points. This talk is based on a joint work with J.J.L.Vel'azquez and Y. Sugiyama.

李朠愛 (Youngae Lee)

Title:
Asymptotic analysis of solutions of the Chern-Simons $CP(1)$ model on a torus

Abstract:
In this talk, we consider a nonlinear elliptic equation derived from the Chern-Simons $CP(1)$ models on a torus.
Recently, it has been known that there might exist nontopological stable entire solutions in $\RN$ under certain conditions.
In view of this fact, we obtain some global condition of the coefficients which is necessary for the equivalence result between stable solutions and topological solution.
Moreover, we also investigate other various bubbling solutions including mountain pass solutions and analyze the asymptotic behavior of general solutions on a torus.

Spreading and vanishing behaviors of radially symmetric solutions in apopulation model with a free boundary

Yuki Kaneko (Waseda University)

Abstract. We discuss free boundary problems for diffusion equations modeling the spreading of new or invasive species, where unknown functions are population density and a spreading front of the species. These problems in multi-dimensions have been studied by Du-Guo (2011, 2012) when a nonlinear function in the diffusion equation is a logistic type. In this talk, we will search for radially symmetric solutions of the problem for general reaction-diffusion equations in a multi-dimensional ball or annulus, and study detailed behaviors of spreading and vanishing in large time.

DEAD-CORE RATES FOR A PARABOLIC EQUATION WITH STRONG ABSORPTION

JONG-SHENQ GUO

We study the dead-core rates for the initial boundary value problem of various parabolic equations with strong absorption. Unlike in many other related problems of singularity formation, we show that the temporal rate of formation of the dead-core is not self-similar for both heat and fast diffusion equations. The proofs rely on self-similar variables and require a delicate use of the Zelenyak method. Moreover, we give some initial data so that the dead-core rate is self-similar for the case of slow diffusion. Some open problems shall also be given.

This talk is based on joint works with Xinfu Chen, Bei Hu, Chia-Tung Ling and Philippe Souplet.

Department of Mathematics, Tamkang University, New Taipei City, Taiwan
E-mail address: jsguo@mail.tku.edu.tw

A priori estimates for Keller-Segel systems

Tomomi Yokota (Tokyo University of Science)

Abstract. In this talk we present how to derive a priori estimates for solutions to several kinds of Keller-Segel systems. More precisely, we deal with the following estimates for respective systems:

- L^p-estimates for degenerate parabolic-parabolic Keller-Segel systems on the whole space.
- Energy estimates for degenerate parabolic-parabolic Keller-Segel systems on bounded domains.
- Uniform-in-time L^p-estimates for parabolic-elliptic Keller-Segel systems with signal-dependent sensitivity.

These estimates yield some new results on the global existence of solutions to respective systems. These are mainly based on joint works with Sachiko Ishida, Takashi Ono, Kentarou Fujie and Michael Winkler.

Removability of time-dependent singularities in the heat equation

Jin Takahashi (Tokyo Institute of Technology, M2)

Abstract. In this talk we discuss solutions of the linear heat equation with time-dependent singularities. First, we show that if a singularity is weaker than the order of the fundamental solution of the Laplace equation, then it is removable. Next, we give an example of a non-removable singularity, which implies the optimality of the condition for removability. We also discuss asymptotics of such non-removable singularities. This is a joint work with Eiji Yanagida (Tokyo Institute of Technology).

The global existence for some double-diffusive convection system with the Homogeneous Neumann boundary condition

Shun Uchida (Waseda University)

Abstract. We consider the initial boundary value problem and the time periodic problem of a system which describes the double-diffusive convection phenomenon between the temparature and the concentration of a solute. For the homogeneous Dirichlet boundary condition, the global existence of these problems is already established. The arguments of proofs rely on the theory of the non-monotone perturbations for subdifferential operators. However, the lack of the coercivity of the Laplacian with the Neumann boundary condition causes some difficulties in this procedure. Especially for the time periodic problem, we need to introduce some approximation procedures for this system with two approximation parameters.

林太家 (Tai-Chia Lin)

Title:
Multiple solutions of steady state PNP-steric systems

Abstract:
The Poisson-Nernst-Planck (PNP) system is an important model to describe ion transport in ionic liquids having many applications in biology, chemistry and physics. The model works well for dilute electrolytes but it should be modified when ionic size effects are considered for crowded ions. This motivates us to study PNP systems with steric effects, called PNP-steric systems. We prove that there are multiple solutions of steady state PNP-steric systems which describe three species ions (two cations and one anion) and one positive constant permanent charge. Our results may be related to the gating and selectivity of ion channels.

A free boundary problem describing cell motility

Harunori Monobe (Meiji University)

Abstract. In this talk, we study a free boundary problem related to cell motility and consider the existence and behavior of global-in-time solutions for the free boundary problem under some boundary conditions. In particular, we consider the behavior of spherically-symmetric solutions and the existence of traveling wave solutions in two-dimensional space. The key point in this analysis is to find an interaction of coefficients which helps us to obtain a priori estimate of the area of time-dependent domains.

ESTIMATES OF THE MEAN FIELD EQUATIONS WITH INTEGRAL SINGULAR SOURCES

Ting-Jung Kuo
(joint work with Chang-Shou Lin)

Abstract: Let M be a compact Riemann surface, $\alpha_j \in \mathbb{N}$ and $h(x)$ be a positive C^2 function of M. In this paper, we consider the following mean field equation:

$$\Delta u(x) + \rho\left(\frac{h(x)\,e^{u(x)}}{\int_M h(x)\,e^{u(x)}} - \frac{1}{|M|}\right) = 4\pi\sum_{j=1}^{d}\alpha_j\left(\delta_{q_j} - \frac{1}{|M|}\right) \text{ in } M.$$

Let u_k be a sequence of blowup solution with ρ_k and $\lim_{k\to\infty}\rho_k = \rho_\infty = 8\pi m$. The analysis is more complicated when α_j is an positive integer. Let p be one of the blow-up points $\{p_1, ..., p_m\}$ of u_k, and $r > 0$ such that in $B_{2r}(p)\setminus\{p\}$, u_k has no blow-up points. We put

$$\rho_{k,p} = \frac{\rho_k\int_{B_r(p)} h(x)\,e^{u_k}dx}{\int_\Omega h(x)\,e^{u_k}dx} \text{ and } \rho_{\infty,p} = \lim_{k\to\infty}\rho_{k,p}.$$

$$u_k(p_{k,i}) = \max_{B_{r_0}(p_i)} u_k(x) \text{ and } \lambda_{k,i} = u_k(p_{k,i}) - \log\int_M h(x)\,e^{u_k}dx$$

The first fundamental question is whether $(\lambda_{k,i} - \lambda_{k,j})$ *is bounded for* $i \neq j$ *or not.* When $\alpha_j \notin \mathbb{N}$ for all j, this question has been answered affirmatively. Our main theorem is to complete this problem if $\alpha_j \in \mathbb{N}$.

Taida Institute for Mathematical Sciences (TIMS)

張覺心 (Chueh-Hsin Chang)

Title :
On the direct and inverse scattering analysis of Camassa-Holm equation

Abstract:
Physical details of Camassa--Holm (CH) equation that are difficult to get in space-time simulation are explored by solving the Lax pair equations within the direct and inverse scattering analysis context.
In the spectral analysis of time-independent Lax equation, both of the continuous and discrete spectrum cases for the initial condition under current investigation are analytically derived. The scattering data derived from the direct scattering transform for the non-reflectionless case are discussed in detail from the physical viewpoint in spectral domain. The evolution of the resulting scattering data in time through the other linear Lax equation and then its inverse scattering transform by Gel'fand-Levitan-Marchenko integral equation is revealed numerically based on an observation by A. Constantin and J. Lenells in J. Nonlinear Math. Phys. 10 (2003), 252-255.

G.Hoshino and T.Ozawa

Title
Analytic smoothing effect for nonlinear Schrödinger equation with quintic nonlinearity

Abstract
We prove the global existence of analytic solutions to nonlinear Schroödinger equation with quintic nonlinearity in n space dimensions for sufficiently small Cauchy data with exponential decay. The smallness assumption on the data is imposed in terms of the critical Sobolev space $\dot{H}^{n/2-1/2}$. Moreover, a characterization of some class of analytic functions is given.

Well-posedness for the Cauchy problem of a system of semirelativistic equations

Kazumasa Fujiwara, Shuji Machihara, Tohru Ozawa

We study time local and global well-posedness of the following Cauchy problem for a system of semirelativistic equations

$$\begin{cases} i\partial_t u + \sqrt{m^2 - \Delta}\, u = \lambda \overline{u} v, \\ i\partial_t v - \sqrt{M^2 - \Delta}\, v = \mu u^2, \\ (u(0), v(0)) = (u_0, v_0), \end{cases} \tag{1}$$

where u, v are complex valued functions of $(t, x) \in \mathbb{R} \times \mathbb{R}$, $\partial_t = \partial/\partial t$, $m, M \in \mathbb{R}$, $\lambda, \mu \in \mathbb{C}$, $\Delta = \partial_x^2 = (\partial/\partial x)^2$ is the Laplacian in $\mathbb{R}$, and $\overline{u}$ is the complex conjugate of u.

Semirelativistic equations with Hartree type nonlinearity are regarded as models of Boson stars. Borgna and Rial studied the Cauchy problem for a single semirelativistic equation with cubic nonlinearity in [1] and they proved the existence of local solutions in H^s with $s > 1/2$, where $H^s = (1 - \Delta)^{-s/2} L^2(\mathbb{R})$ is the usual Sobolev space. The method of proof depends essentially on the Sobolev embedding $H^s \hookrightarrow L^\infty$.

In the case where $0 \le s \le 1/2$, however, the method loses its meaning because the uniform control by H^s norm breaks down. Moreover, we remark that Strichartz type estimates are not sufficient for a contraction argument unless the uniform control by H^s norm is available. Therefore, it is natural to introduce the Bourgain method to study (1) in H^s with $0 \le s \le 1/2$.

We state our main results. We introduce notation of the maximal existence time of solutions. Let $s \ge 0$ and let $(u_0, v_0) \in H^s \times H^s$. Then for any s' with $0 \le s' \le s$, we define

$$T(s') = \sup \left\{ T > 0 \, ; (u, v) \in C([0, T]; H^{s'}) \text{ is a solution of (1) } \right\}.$$

Our results are the following.

Theorem 1

Let $s \ge 0$ and let $(u_0, v_0) \in H^s \times H^s$. Then there exists $T > 0$ and a unique pair of solutions $(u, v) \in C([0, T], H^s \times H^s)$ to (1). Moreover, $T(s) = T(0)$.

Theorem 2

Let λ and μ satisfy $\lambda = c\overline{\mu}$ with some constant $c > 0$. Let $s \ge 0$. Then the solutions of Theorem 1 extend globally and satisfy $(u, v) \in C(\mathbb{R}; H^s \times H^s) \cap L^\infty(\mathbb{R}; L^2 \times L^2)$.

We prove Theorem 1 by a contraction argument based on the Bourgain norm in $X^{s,b}$ defined as

$$\|u : X^{s,b}_{m,\pm}\| = \left\| \langle \xi \rangle^s \left\langle \tau \pm \sqrt{m^2+\xi^2} \right\rangle^b \widetilde{u}(\tau,\xi) : L^2_\tau L^2_\xi \right\|,$$

$$\|u : X^{s,b}_{m,\pm}[T_0, T_0+T]\|$$
$$= \inf \left\{ \left\| u' : X^{s,b}_{m,\pm} \right\| \ ; \ \begin{array}{l} u'(t,x) = u(t,x) \text{ on } [T_0, T_0+T] \times \mathbb{R}, \\ \text{supp } u' \subset [T_0-2T, T_0+2T] \times \mathbb{R} \end{array} \right\}.$$

We also use the auxiliary norm in Y^s defined as

$$\|u : Y^{s}_{m,\pm}\| = \left\| \langle \xi \rangle^s \left\langle \tau \pm \sqrt{m^2+\xi^2} \right\rangle^{-1} \widetilde{u} : L^2_\xi L^1_\tau \right\|,$$

$$\|u : Y^{s}_{m,\pm}[T_0, T_0+T]\|$$
$$= \inf \left\{ \left\| u' : Y^{s}_{m,\pm} \right\| \ ; \ \begin{array}{l} u'(t,x) = u(t,x) \text{ on } [T_0, T_0+T] \times \mathbb{R}, \\ \text{supp } u' \subset [T_0-2T, T_0+2T] \times \mathbb{R} \end{array} \right\}$$

especially for the critical case $s = 0$. We give several types of bilinear and trilinear estimates by means of those norms by the method of [3], which are applied to the arguments of the well-posedness and the persistence of regularity $T(s) = T(0)$.

Under the constraint $\lambda = c\overline{\mu}$, we show the following conservation law of charge, namely, the conservation law of the L^2 norm :

$$\|u(t) : L^2\|^2 + c\|v(t) : L^2\|^2 = \|u_0 : L^2\|^2 + c\|v_0 : L^2\|^2 \tag{2}$$

for any $t \in \mathbb{R}$. For the proof, we apply the argument by one of us [2], which need not take smooth approximation of solutions. Our proof uses only a weak regularity of solutions guaranteed in $X^{0,1/2}_- \times X^{0,1/2}_+$, which makes sense all the calculations for (2) on the basis of the trilinear estimates. Then we have the global existence of the solutions in L^2, and also $H^s, s > 0$, since $T(s) = T(0)$ in Theorem 1.

References

[1] J. P. Borgna and D. F. Rial, "Existence of ground states for a one-dimensional relativistic Schrödinger equation", J. Math. Phys, **53** 062301 (2012), http://dx.doi.org/10.1063/1.4726198.

[2] T. Ozawa, "Remarks on proofs of conservation laws for nonlinear Schrödinger equations", Calc. Var. Partial Differential Equations, **25**(2006), 403–408.

[3] S. Selberg and A. Tesfahun, "Low regularity well-posedness of the Dirac-Klein-Gordon equations in one space dimension", Commun. Contemp. Math. **10**(2008), 181–194.