Critical mass of degenerate Keller-Segel system with no-flux and Neumann boundary conditions

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We consider the time-global existence of solutions to the following parabolic system:

$$\begin{cases} \partial_t u = \nabla \cdot (\nabla u^m - u\nabla v), & x \in \Omega,\ t > 0, \\ \partial_t v = \Delta v - \gamma v + u, & x \in \Omega,\ t > 0, \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x), & x \in \Omega, \end{cases} \tag{KS}$$

where γ is a non-negative constant and Ω is a bounded domain in $\mathbb{R}^d$, $d > 2$, with smooth boundary. We focus on the critical exponent $m = 2 - 2/d$ and impose the no-flux and Neumann boundary conditions for u and v, respectively. Notice that the mass of u over Ω is preserved by the no-flux boundary condition. One of the fundamental questions concerning the system (KS) is whether or not a threshold mass of u that separates the existence from non-existence of blow-up solutions exists. Therefore we are interested in the largest mass of u such that the time-global existence of solutions is guaranteed. We seek such a mass from view point of the minimizing movement scheme, which is a method of variational construction of solutions to gradient flows. We show the least upper bound M_N of the mass such that time-global solutions to (KS) can be constructed by the minimizing movement scheme and the time-global existence of solutions to (KS) under the assumption that $\|u_0\|_{L^1} < M_N$.