Transient growth phenomenon in a parabolic-elliptic chemotaxis system

Michael Winkler

Paderborn University, Germany

We consider variants of the Keller-Segel system of chemotaxis which contain logistic-type source terms and thereby account for proliferation and death of cells. We briefly review results and open problems with regard to the fundamental question whether solutions exist globally in time or blow up. The primary focus will then be on the prototypical parabolic-elliptic system

$$\begin{cases} u_t = \varepsilon u_{xx} - (uv_x)_x + ru - \mu u^2, \\ 0 = v_{xx} - v + u, \end{cases}$$

in bounded real intervals. The corresponding Neumann initial-boundary value problem, though known to possess global bounded solutions for any reasonably smooth initial data, is shown to have the property that the so-called *carrying capacity* $\frac{r}{\mu}$ can be exceeded dynamically to an arbitrary extent during evolution in an appropriate sense, provided that $\mu < 1$ and that $\varepsilon > 0$ is sufficiently small. To achieve this, an analysis of the hyperbolic-elliptic problem obtained on taking $\varepsilon \to 0$ is carried out; indeed, it turns out that the latter limit problem possesses some solutions which blow up in finite time with respect to their spatial L^∞ norm.

This result is in stark contrast to the case of the corresponding Fisher-type equation obtained upon dropping the term $-(uv_x)_x$, and hence reflects a drastic peculiarity of destabilizing action due to chemotactic cross-diffusion, observable even in the simple spatially one-dimensional setting. Numerical simulations underline the challenge in the analytical derivation of this result by indicating that the phenomenon in question occurs at intermediate time scales only, and disappears in the large time asymptotics.