Anomalous enstrophy dissipation and 2D vortex collapse

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In 2D turbulence, there emerges the inertial range in the energy density spectrum of the flow that corresponds to the forward cascade of enstrophy for sufficiently small viscosity[1, 4, 5]. It is pointed out that this phenomenon is caused by the enstrophy dissipation in the inviscid limit of the incompressible flow field. However, since smooth solutions of the 2D Euler equations conserve the enstrophy, one can expect that non-smooth weak solutions of the 2D Euler equations play an important role in the realization of this anomalous enstrophy dissipation. Eyink[2] have shown that weak solutions in $L^p(\mathbb{R}^2)$ did not dissipate the enstrophy for $1 < p < \infty$ in a weak sense. Therefore, we consider the 2D Euler equations for the initial vorticity data in the space of Radon measure on $\mathbb{R}^2$ to obtain weak solutions with the enstrophy dissipation. When we assume that the vorticity field is represented by a δ measure whose support consists of N discrete points, the 2D Euler equations are reduced to the ordinary differential equations for these N singular points, which is called the *point vortex system*. It is known that there exists a singular solution of this system, called *self-similar collapse*, in which the point vortices collapse self-similarly to a point in finite time[3]. We here want to investigate whether or not this singular solution yields the anomalous dissipation of the enstrophy at the event of collapse. However, we are unable to deal with this singular solution as it is, since it cannot be extended globally in time beyond the critical time. We thus consider the 2D Euler-α equations[6], which is a dispersive regularization of the 2D Euler equations, for the point-vortex vorticity data, which we refer to as the *α-point vortex (αPV) system*. Mathematically, since Oliver and Shkoller[6] have established the global existence of a unique weak solution of the 2D Euler-α equations for the initial vorticity data in the space of Radon measure on $\mathbb{R}^2$ in the sense of distribution, one can construct a solution of αPV system globally in time. Then, as the $\alpha \to 0$ limit of the global solution of the αPV system, we are able to extend this collapsing solution of the 2D Euler equations. In this talk, we first consider the three αPV system and show that, in the $\alpha \to 0$ limit, the self-similar collapse is connected with the self-similar expanding solution and that the anomalous enstrophy dissipation occurs at the critical time in the sense of distribution[7]. We also show some numerical results on the anomalous enstrophy variations owing to singular collapsing solutions of more point vortices.

References

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