Liouville theorems for scaling invariant superlinear parabolic problems with gradient structure

Pavol Quittner

Comenius University, Slovakia

We provide a simple method for obtaining new Liouville theorems for scaling invariant superlinear parabolic problems with gradient structure. To illustrate the method we prove Liouville theorems (guaranteeing nonexistence of positive classical solutions) for the following model problems: the scalar nonlinear heat equation

$$u_t - \Delta u = u^p \qquad \text{in } \mathbb{R}^n \times \mathbb{R},$$

its vector-valued generalization with a p-homogeneous nonlinearity and the linear heat equation in $\mathbb{R}^n_+ \times \mathbb{R}$ complemented by nonlinear boundary conditions of the form $\partial u/\partial \nu = u^q$. Here ν denotes the outer unit normal on the boundary of the halfspace $\mathbb{R}^n_+$ and the exponents $p, q > 1$ satisfy $p < n/(n-2)$ and $q < (n-1)/(n-2)$ if $n > 2$ (or $p < (n+2)/(n-2)$ and $q < n/(n-2)$ if $n > 2$ and some symmetry of the solutions is assumed). As a typical application of our nonexistence results we provide blow-up rate estimates for positive solutions of related problems in bounded and unbounded domains.