Threshold solutions for semilinear heat equations with polynomial decay initial data

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We consider asymptotic behavior of global positive solutions of the Cauchy problem for the semilinear heat equation

$$\begin{cases} u_t = \Delta u + u^p, & x \in \mathbf{R}^N,\ t > 0, \\ u(0,x) = u_0(x), & x \in \mathbf{R}^N. \end{cases}$$

Let $u_0(x) = \alpha\phi(x)$, where α is a parameter and ϕ is a fixed function with suitable decay as $|x| \to \infty$. Then there exists $\alpha^* \in (0,\infty)$ such that the solution exists globally if and only if $\alpha \le \alpha^*$. Kavian (1987) and Kawanago (1996) observed the difference between the behavior of threshold solution ($\alpha = \alpha^*$) and sub-threshold solution ($\alpha < \alpha^*$) in the case $p < (N+2)/(N-2)$ provided ϕ has an exponential decay. Later, Quittner (2008) and Poláčik and Quittner (2008) studied the problem involving the case $p = (N+2)/(N-2)$ assuming that ϕ satisfies

$$\lim_{x\to\infty} |x|^{2/(p-1)}\phi(x) = 0.$$

It should be mentioned that the polynomial rate $|x|^{-2/(p-1)}$ at $|x| = \infty$ is the borderline decay of $u_0(x)$ for the existence of global solutions. In this talk, we study the Cauchy problem with initial data those have the polynomial decay at $|x| = \infty$, and discuss the behavior of global solutions by means of the potential well with forward self-similar transformation. In particular, we are interested in the behavior of the threshold solutions between global existence and blow-up.