A Keller-Segel model with critical mass in any space dimension

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Denoting ρ the cell density and c the chemoattractant concentration, we study radial solutions in a ball $B \subset \mathbb{R}^N$ of the following chemotaxis model

$$\rho_t = \Delta\rho - \nabla[\rho^q \, \nabla c]$$
$$-\Delta c = \rho$$

with no flux and Dirichlet conditions on ∂B respectively for ρ and c. We focus on the critical exponent

$$q = \frac{2}{N}$$

which makes this Keller-Segel system a kind of generalization of the "linear" case $q = 1$ in dimension $N = 2$, the latter being well-known for its critical mass 8π. In dimension $N \geq 3$, we show that this system also exhibits a critical mass phenomenon, but with strong qualitative differences, especially for the critical mass case. Moreover, this evolution equation can formally be seen as a gradient flow on an "infinite dimensional Riemannian manifold". In the subcritical case, this inspired a proof that the uniform convergence of radial solutions toward the unique steady state has exponential speed. This result is valid and new even for the most studied case $N = 2$.