Singular limit of a stochastic Allen-Cahn equation

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This talk is concerned with an Allen-Cahn type diffusion equation on a domain in $\mathbb{R}^n$ with an ε-dependent mild noise. More precisely, I will consider an equation of the form

$$u_t = \Delta u + \varepsilon^{-2} f(u) + \varepsilon^{-1} \xi_t^{\varepsilon}.$$

Here, f is a balanced bistable nonlinearity, and ξ_t^{ε} is a smooth but random function of t of order $O(\varepsilon^{-\gamma})$ for some constant $0 < \gamma < 1/3$ that behaves like white noise as $t \to +0$. The initial data $u_0(x)$ is independent of ε. We show that a steep internal layer develops withing a very short time of order $O(\varepsilon \ln \varepsilon^{-1})$ (wich we call the "generation of interface"), and derive the law of motion of the interface in the singular limit $\varepsilon \to +0$ (or the sharp interface limit), which is given in the form

$$V = \kappa + (c_0 \alpha) \dot{w}_t,$$

where c_0, α are some positive constants and $\dot{w}_t$ is a white noise in t. This reulst extends the work of Funaki (1999) and Weber (2010), who derived the same law of motion for a very special class of ε-dependent initial data without discussing the generation of interface. This is joint work of Dimitra Antonopoulou and Georgia Karali.