$\mathcal{D}$-solutions of the stationary Navier-Stokes equations past an obstacle

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This is the joint work with Horst Heck(Bern Univ., Switzerland) and Hyunseok Kim(Sogan Univ., Korea).

In an exterior domain $\Omega \subset \mathbb{R}^3$, we consider the stationary Navier-Stokes equations

$$\text{(N-S)} \qquad \begin{cases} -\Delta u + u \cdot \nabla u + \nabla p = \operatorname{div} F \quad \text{in } \Omega, \\ \operatorname{div} u = 0 \quad \text{in } \Omega, \\ u = 0 \quad \text{on } \partial\Omega, \\ u(x) \to u^\infty \quad \text{as } |x| \to \infty \end{cases}$$

where F and u^∞ are the given 3×3-tensors of functions on Ω and the prescribed constant vector in $\mathbb{R}^3$ at infinity, respectively. For a family $\{F_n, u_n^\infty\}_{n=1}^\infty$ of given data, we denote by $\{u_n\}_{n=1}^\infty$ the family of $\mathcal{D}$-solutions corresponding to (N-S). In this talk, we shall show that there is a constant $\delta > 0$ such that if $\|F_n\|_{L^{3/2,\infty}} + |u_n^\infty| \leqq \delta$ holds for all $n = 1, 2, \cdots$ and if $F_n \rightharpoonup F$ weakly-$*$ in $L^{3/2,\infty}(\Omega) \cap L^2(\Omega)$, $u_n^\infty \to u^\infty$ in $\mathbb{R}^3$ as $n \to \infty$, then we have that $\nabla u_n \rightharpoonup \nabla u$ weakly-$*$ in $L^{3/2,\infty}(\Omega) \cap L^2(\Omega)$, where u is a unique $\mathcal{D}$-solution of (N-S). Furthermore, if $F_n \to F$ strongly in $L^2(\Omega)$, then it holds that $\nabla u_n \to \nabla u$ strongly in $L^q(\Omega)$ for all $3/2 < q \leqq 2$. This result may be regarded as generalization of Shibata-Yamazaki [2] since we may include such a critical case as $u^\infty = 0$. It should be noted that the solution u of (N-S) with $u^\infty = 0$ behaves completely different from those of the case $u^\infty \neq 0$.

References

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[2] Shibata, Y., Yamazaki, M., Uniform estimates in the velocity at infinity for stationary solutions to the Navier-Stokes exterior problem. Jpn. J. Math. (N.S.) **31**, 225–279 (2005).