Heat equation with a nonlinear boundary condition and uniformly local L^r spaces

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This is joint work with my student Ryuichi Sato (Tohoku University) and it is concerned with the heat equation with a nonlinear boundary condition,

$$\begin{cases} \partial_t u = \Delta u, & x \in \Omega,\ t > 0, \\ \nabla u \cdot \nu(x) = |u|^{p-1}u, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = \varphi(x), & x \in \Omega, \end{cases} \tag{P}$$

where $N \geq 1$, $p > 1$, Ω is a smooth domain in $\mathbf{R}^N$, $\partial_t = \partial/\partial t$ and $\nu = \nu(x)$ is the outer unit normal vector to $\partial\Omega$. In this talk we assume that the initial function belongs to a suitable uniformly local L^r spaces and establish the local existence and the uniqueness of the solutions of (P). Our analysis enables us to obtain the sharp lower estimates of the blow-up time of the solutions with the initial data $\lambda\psi$ as $\lambda \to 0$ or $\lambda \to \infty$ and the lower blow-up estimates of the solutions.