Non-existence results for the semilinear Schrödinger equation

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This talk is based on our recent works [1, 2] with Takahisa Inui in Kyoto University. We consider the following Cauchy problem for the semilinear Schrödinger equation:

$$\begin{cases} (i\partial_t + \Delta)u = \mu|u|^p, & (t,x) \in [0,T_\lambda) \times \mathbb{R}^d, \\ u(0,x) = \lambda f(x), & x \in \mathbb{R}^d, \end{cases} \tag{NLS}$$

where $u = u(t,x)$ is a $\mathbb{C}$-valued unknown function, $\mu \in \mathbb{C}\backslash\{0\}$, $p > 1$, $\lambda \geq 0$, $f = f(x)$ is a $\mathbb{C}$-valued given function and T_λ is a maximal existence time of the solution.

Our first aim is to give a small data blow-up result for (NLS) in the L^2-subcritical case $p < 1 + 4/d =: p_0$. More precisely, we prove that in the case of $1 < p < p_0$ and $s \geq 0$, for a suitable H^s-function f, there exists $\lambda_0 > 0$ such that for any $\lambda \in (0, \lambda_0)$ such that the following estimates hold:

$$T_\lambda \leq C_0 \lambda^{-\kappa_0} \quad \text{and} \quad \lim_{t \to T_\lambda - 0} \|u(t)\|_{H^s} = \infty,$$

where $\kappa_0, C_0 > 0$ are constants independent of λ.

Our second aim is to prove a large data blow-up result for (NLS) in the H^s-critical case $p = p_s =: 1 + 4/(d-2s)$ or the H^s-subcritical case $p < p_s$, for some $s \geq 0$. More precisely, we show that in the case $1 < p \leq p_s$, for a suitable H^s-function f, there exists $\lambda_1 > 0$ such that for any $\lambda > \lambda_1$, the following estimates hold:

$$T_\lambda \leq C_s \lambda^{-\kappa_s} \quad \text{and} \quad \begin{cases} \lim_{t \to T_\lambda - 0} \|u(t)\|_{H^s} = \infty, & (\text{if } 1 < p < p_s), \\ \|u\|_{L_t^r(0,T_\lambda; B^s_{\rho,2})} = \infty, & (\text{if } p = p_s), \end{cases}$$

where $\kappa_s, C_s > 0$ are constants independent of λ and (r, ρ) is an admissible pair.

Our final aim is to prove non-existence local weak solution for (NLS) in the H^s-supercritical case $p > p_s$, which means that for $p > p_s$, there exists a function $f \in H^s$ such that if there exist $T > 0$ and a weak solution u to (NLS) on $[0, T)$, then $\lambda = 0$.

We draw the table for the results of only H^1-solution.

<table>
<tr><td>$f \setminus p$</td><td>1</td><td></td><td>$1+4/d$</td><td></td><td>$1+4/(d-2)$</td><td></td></tr>
<tr><td rowspan="2">H^1</td><td rowspan="2"></td><td colspan="4">Large data blow-up</td><td rowspan="2">Non-existence local of weak sol.</td></tr>
<tr><td>Small data blow-up</td><td colspan="3">Small data global well-posedness</td></tr>
</table>

References

[1] M. Ikeda, T. Inui, *Small data blow up of L^2 or H^1-solution for the semilinear Schrödinger equation without gauge invariance*, preprint.

[2] M. Ikeda, T. Inui, *Some non-existence results for the semilinear Schrödinger equation without gauge invariance*, in preparation.