On a non-blow up criterion involving vorticity direction under the non-slip boundary condition for the three-dimensional Navier-Stokes flow

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We give a geometric non-blow up criterion on the direction of the vorticity for the three-dimensional half-space Navier-Stokes flow under the non-slip boundary condition whose initial data is just bounded and may have infinite energy. We prove that under a restriction on behavior in time (type I condition) the solution does not blow up if the vorticity direction is uniformly continuous at the place where the vorticity is large even if we impose the Dirichlet boundary conditions.

A similar geometric regularity criterion for non-blow up has been proved by P. Constantin and C. Fefferman (1993) under Lipschitz regularity condition for the whole space has been established by H. Miura and the author (2011), and for the half space with the slip boundary condition. Their argument does not directly apply to the non-slip boundary condition since a key Liouville result for the two dimensional flow does not directly extend to the case of non-slip boundary.

We apply a representation formula for the vorticity (Y. Maekawa (2012)) and establish a Liouville type result under the non-slip boundary condition for type I blow-up. This enables us to prove that a continuous alignment condition for the vorticity prevents the blow-up even under the non-slip boundary condition which may produce a lot of vorticity near the boundary.

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