Effect of higher order derivatives of initial data to the blow-up set for a semilinear heat equation

Yohei Fujishima

Graduate School of Engineering Science, Osaka University, Japan

We consider the blow-up problem for a semilinear heat equation with large initial data

$$\begin{cases} \partial_t u = \Delta u + u^p, & x \in \Omega, \ \ t > 0, \\ u(x,t) = 0, & x \in \partial\Omega, \ \ t > 0, \\ u(x,0) = \lambda + \varphi(x), & x \in \Omega, \end{cases} \tag{P}$$

where $p > 1$, $N \geq 1$, $\Omega \subset \mathbf{R}^N$ is a domain in $\mathbf{R}^N$, $\lambda > 0$ is a sufficiently large constant and $\varphi \in C_0^\infty(\Omega)$ is a nonnegative function. For problem (P) with initial data replaced by $u(x,0) = \lambda\varphi(x)$, it has been proved that the solution blows up near the maximum points of φ of which $|\Delta\varphi|$ is small if λ is sufficiently large. In particular, if the set of maximum points of φ consists of two points α and β and $|\Delta\varphi(\alpha)| < |\Delta\varphi(\beta)|$, then the solution blows up only near the point α. However, it seems difficult to obtain further information of the blow-up set for problem (P) with $u(x,0) = \lambda\varphi(x)$, and we can not characterize the location of the blow-up set for the case where $|\Delta\varphi(\alpha)| = |\Delta\varphi(\beta)|$. In this talk, we consider problem (P) with sufficiently large λ, and prove that the blow-up set is characterized by $|\Delta^2\varphi|$ at maximum points of φ for the case where $|\Delta\varphi(\alpha)| = |\Delta\varphi(\beta)|$.