Extinction of solutions of the fast diffusion equation with a critical exponent

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We consider the problem

$$\begin{cases} u_t = \nabla \cdot (u^{m-1} \nabla u), & x \in \mathbb{R}^n, \ t > 0, \\ u(x,0) = u_0(x) \geq 0, & x \in \mathbb{R}^n, \ u_0 \not\equiv 0. \end{cases}$$

It is well known that conservation of mass holds if and only if $m \geq (n-2)/n$, and for $m < (n-2)/n$ extinction of solutions in finite time occurs for initial data which decay fast enough. This means that for a solution u emanating from such initial data there exists an extinction time $T = T(u_0) \in (0, \infty)$ such that $u > 0$ in $\mathbb{R}^n \times (0, T)$ and $u(\cdot, T) \equiv 0$.

We shall discuss results on the asymptotic behavior of solutions near the extinction time when

$$m = \frac{n-4}{n-2}, \qquad n > 2.$$

These results were obtained in collaboration with John R. King, Hannes Stuke and Michael Winkler.