Blowup and global existence for some complex Ginzburg-Landau equations

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In this talk I will discuss some recent work in collaboration with S. Correia, F. Dickstein and F. B. Weissler on the family of complex Ginzburg-Landau equation $u_t = e^{i\theta}\Delta u + e^{i\gamma}|u|^\alpha u$ set on the whole space, where $-\frac{\pi}{2} \le \theta, \gamma \le \frac{\pi}{2}$. In the particular case $\gamma = \theta$, we prove finite-time blowup when the initial value satisfies some energy condition, and study the behavior of the blow-up time when $\theta \to \pm\frac{\pi}{2}$, i.e., in the NLS limit. In another special case, namely for $\theta = \pm\frac{\pi}{2}$ and $-\frac{\pi}{2} < \gamma < \frac{\pi}{2}$, we prove that $\alpha = \frac{N}{2}$ is a critical exponent of Fujita type. I will mention a number of related open questions. Finally, I will briefly discuss the existence of standing waves for the more general equation $u_t = e^{i\theta}\Delta u + e^{i\gamma}|u|^\alpha u + ku$, set either on the whole space, or on a bounded domain with Dirichlet boundary conditions.