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Nonnegative solutions of semilinear elliptic equations in half-spaces

Carmen Cortázar
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This is a joint work with Manuel Elgueta from P. Universidad Católica de Chile and Jorge García-Melián from Universidad de La Laguna.

We study the monotonicity of solutions of the semilinear elliptic problem

$$\begin{cases} -\Delta u = f(u) & \text{in } \mathbb{R}^N_+ \\ u = 0 & \text{on } \partial\mathbb{R}^N_+ \end{cases}$$

where the nonlinearity f is assumed to be C^1 and globally Lipschitz with $f(0) < 0$, and $\mathbb{R}^N_+ = \{x \in \mathbb{R}^N : \ x_N > 0\}$ stands for the half-space.

Blow up for harmonic map flow

Juan Dávila
Universidad de Chile

We study singularity formation for the harmonic map flow from a two dimensional domain into the sphere. We show that for suitable initial conditions the flow develops a type 2 singularity at some point in finite time, and that this is stable under small perturbations of the initial condition. This phenomenon and the rate of blow up were studied formally by van den Berg, Hulshof and King (2003) and proved by Raphael and Schweyer (2013) in the class of radial and 1-corrotationally symmetric maps. Our results hold without any symmetry assumptions.

Bubbling in the critical heat equation: the role of Green's function

Manuel del Pino
Universidad de Chile

We investigate the point-wise, infinite-time bubbling phenomenon for positive solutions of the semilinear heat equation at the critical exponent in a bounded domain. We build an invariant manifold for the flow which ends at k bubbling points of the domain for any given k. The delicate role of dimension is described. This is joint work with C. Cortázar and M. Musso.

Multiplicity results for sign changing bound state solutions of a semilinear equation

Marta García-Huidobro
Pontificia Universidad Católica de Chile

(Joint work with C. Cortázar and P. Herreros)

In this talk we give conditions on the nonlineary f so that the problem

$$\begin{aligned} &\Delta u + f(u) = 0, \quad x \in \mathbb{R}^N, \ N \geq 2, \\ &\lim_{|x| \to \infty} u(x) = 0, \end{aligned} \tag{1}$$

has at least two solutions having a prescribed number of nodal regions and for which $u(0) > 0$. Any nonconstant solution to (1) is called a bound state solution. Bound state solutions such that $u(x) > 0$ for all $x \in \mathbb{R}^N$, are referred to as a first bound state solution, or a ground state solution. The existence of ground states for (1) has been established by many authors under different regularity and growth assumptions on the nonlinearity f, both for the Laplacian operator and the degenerate Laplacian operator, see for example [1, 2], [3] and [6] in the case of a regular f ($f \in C[0, \infty)$) for the case of the semilinear equation, and [7], [8] and [5] for both the singular and regular case in the quasilinear situation. The main assumptions on the nonlinearity f are

(f_1) f is a continuous function defined in $\mathbb{R}$, and f is locally Lipschitz in $\mathbb{R} \setminus \{0\}$.

(f_2) There exists $\delta > 0$ such that if we set $F(s) = \int_0^s f(t)dt$, it holds that $F(s) < 0$ for all $0 < |s| < \delta$, and $\lim\limits_{s \to -\infty} F(s) = \lim\limits_{s \to \infty} F(s)$, $F(s) < \lim\limits_{s \to \infty} F(s)$ for all $s \in \mathbb{R}$.

(f_3) F has a local maximum at some $\gamma \in (\delta, \infty)$ and $F(\gamma) > 0$.

(f_4) there exists $\theta \in (0, 1)$ such that

$$\lim_{s \to \infty} \Big(\inf_{s_1, s_2 \in [\theta s, s]} Q(s_2) \Big(\frac{s}{f(s_1)} \Big)^{N/2} \Big) = \infty,$$

where $Q(s) := 2NF(s) - (N-2)sf(s)$.

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Multiplicity of solutions for an elliptic equation with a singular nonlinearity and a gradient term

Ignacio Guerra
Universidad de Santiago de Chile

We consider the problem

$$\begin{aligned} -\Delta u &= \lambda \frac{(1+|\nabla u|^q)}{(1-u)}, \quad 0<u<1, \quad \text{in} \quad B, \\ u &= 0 \qquad \text{on} \quad \partial B, \end{aligned}$$

where B is the unit ball in $\mathbb{R}^N$, $q \geq 0$ and $\lambda \geq 0$.

The problem with $q = 0$ is well know. In fact, Joseph & Lundgren found that for $2 < N < 4+2\sqrt{2}$ there are infinitely many solutions for some $\lambda = \lambda_* > 0$. On the other hand, they also found that for $N > 4 + 2\sqrt{2}$ there exists λ^* such that there exists a unique solution for each $0 < \lambda < \lambda^*$.

Here we study the existence of solutions for this problem when $q > 0$. In particular, we found a range of q and N where there exists $\lambda_* > 0$ such that are infinitely many solutions for $\lambda = \lambda_*$.

Simultaneous and nonsimultaneous blow-up for a semilinear parabolic system in higher dimension

Junichi Harada
Akita University

We discuss the possibility of simultaneous and nonsimultaneous blow-up for a semilinear parabolic system: $z_t = \Delta z + z^2$, where z is a complex valued function. We construct two types of blow-up solutions. One is an ODE type blow-up solution, and another is a non self-similar blow-up solution. We explain how the matched asymptotic expansion is derived in a parabolic system case.

Minimal solutions of a semilinear elliptic equation with a dynamical boundary condition

Kazuhiro Ishige
Tohoku University

This is a joint work with Marek Fila (Comenius University, Slovakia) and Tatsuki Kawakami (Osaka Prefecture University, Japan).

We study properties of positive solutions of a semilinear elliptic equation with a linear dynamical boundary condition. We establish the semigroup property for minimal solutions, show that every local-in-time solution can be extended globally, and reveal a relationship between minimal solutions of the time-dependent problem and minimal stationary solutions.

Metastability results for a quasi-linear parabolic equation with drift

Hitoshi Ishii
Waseda University

I present metastability results for quasi-linear parabolic PDE with a drift term. M. Freidlin and L. Koralov (ArXiv:0903.0428v2(2012), Probab. Theory Related Fields (2000)) have investigated the asymptotic behavior of solutions $u^\varepsilon(x,t)$, as $(t,\varepsilon)\to(\infty,0)$, of the quasi-linear parabolic PDE

$$u_t^\varepsilon(x,t)=\sum_{i,j=1}^n a_{ij}(x,u^\varepsilon(x,t))u_{x_ix_j}^\varepsilon(x,t)+\sum_{i=1}^n b_i(x)u_{x_i}^\varepsilon(x,t) \quad \text{in } \; Q:=\Omega\times(0,\,\infty).$$

with the Dirichlet condition $u^\varepsilon(x,t)=g(x)$ on the parabolic boundary $\partial_{\mathrm{p}}Q$, where Ω is a smooth bounded domain in $\mathbb{R}^n$, and they have established some results by the probabilistic methods. The results are concerned with the asymptotic behavior of $u^\varepsilon(x,t)$ in the logarithmic time scale, that is, the behavior of $u^\varepsilon(x,\exp(\lambda/\varepsilon))$ as $\varepsilon\to0$ for each fixed $\lambda>0$. I explain a PDE approach to the same asymptotic problem, which is based on joint work with P. E. Souganidis of the University of Chicago.

On the Large time behavior of small solutions for the critical Burgers equation

Tsukasa Iwabuchi
Osaka City University

We consider the Cauchy problem for the critical Burgers equation in the Besov spaces. In the Besov space $\dot{B}^0_{\infty,1}(\mathbb{R}^n)$, we show the existence of global solutions, which are bounded in time, for small initial data. We also consider the large time behavior of the solutions to show that the solution behaves like the Poisson kernel with the initial data in $L^1(\mathbb{R}^n)$ and small in $\dot{B}^0_{\infty,1}(\mathbb{R}^n)$.

Symmetric solutions of semilinear elliptic equations

Ryuji Kajikiya

Saga University

We study the existence of partially symmetric positive solutions of the generalized Hénon equation

$$-\Delta u = f(x)u^p, \quad u > 0 \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega.$$

Here Ω is a bounded domain in $\mathbb{R}^N$, $1 < p < \infty$ when $N = 2$, $1 < p < (N+2)/(N-2)$ when $N \geq 3$, $f \in L^\infty(\Omega)$ and $f(x)$ may be positive or may change its sign. Let G be a closed subgroup of the orthogonal group, H a closed subgroup of G, $f(x)$ and Ω are G invariant. Then we show the existence of a positive solution which is H invariant but G non-invariant under suitable assumptions of H, G and $f(x)$. Applying the theorem to various symmetric domains, we find many positive symmetric solutions.

Solutions with prescribed number of zeros to a nonlinear elliptic equation with weights on $\mathbb{R}^d$

Rául Manásevich
Universidad de Chile

We consider the existence of radial sign-changing solutions, with prescribed number of zeros for the problem

$$\operatorname{div}\big(a\nabla u\big) + bf(u) = 0\,, \qquad \lim_{|x|\to+\infty} u(x) = 0$$

where a and b are two positive, radial, smooth functions defined on $\mathbb{R}^d \setminus \{0\}$, with $f \in C(\mathbb{R})$, and satisfying some additional conditions. We give a proof that also holds for solutions with compact support unifying in this form previous results. The results can be extended to the p-laplace operator with weight.

This talk is based on a work in cooperation with Carmen Cortazar, Jean Dolbeault and Marta Garcia Huidobro.

Structure of positive solutions for semilinear elliptic equations with supercritical growth

Yasuhito Miyamoto

University of Tokyo

Let $B \subset \mathbb{R}^N$, $N \geq 3$, be a unit ball. We study the shape of the bifurcation curve of the positive solutions to the supercritical elliptic equation in B

$$\begin{cases} \Delta u + \lambda f(u) = 0 & \text{in} \quad B, \\ u = 0 & \text{on} \quad \partial B, \\ u > 0 & \text{in} \quad B, \end{cases}$$

where $f(u) = u^p + g(u)$, $p > p_S := (N+2)/(N-2)$, and $g(u)$ is a lower order term. This problem has a singular solution (λ^*, u^*). In this talk we classify the bifurcation curves with the number of the turning points T. The Joseph-Lundgren exponent

$$p_{JL} := \begin{cases} 1 + \dfrac{4}{N - 4 - 2\sqrt{N-1}} & (N \geq 11), \\ \infty & (2 \leq N \leq 10) \end{cases}$$

plays an important role. We show the following: We assume that f satisfies certain assumptions. If $p_S < p < p_{JL}$, then $m(u^*) = \infty$ and $T = \infty$. If $p \geq p_{JL}$, then $m(u^*) < \infty$. In this case, if $m(u^*) = 0$, then $T = 0$, and hence a classical positive solution is unique when it exists. If $m(u^*) \geq 1$, then $m(u^*) \leq T < \infty$. In particular if $N \geq 11$, $p \geq p_{JL}$, $m(u^*) \geq 1$, and u^* is nondegenerate, then the last case occurs. A typical example is $f(u) = (u+1)^p + b(u+1)^q$, $p_S < q < p_{JL} \leq p$. Main tools are the intersection number, a singular Sturm-Liouville theory, and a phase plane analysis

The talk is based on a joint work with Yuki Naito of Ehime University.

Dispersive estimates for rational symbols and local well-posedness of the nonzero energy Novikov-Veselov equation

Claudio Muñoz
Universidad de Chile

We consider the Cauchy problem for the two-dimensional Novikov-Veselov equation integrable via the inverse scattering problem for the Schrödinger operator with fixed negative energy. The associated linear equation is characterized by a rational symbol which is not a polynomial, except when the energy parameter is zero. With the help of a complex analysis point of view of the problem, we establish uniform decay estimates for the linear solution with gain of almost one derivative, and we use this result together with Fourier decomposition methods and $X^{s,b}$ spaces to prove local well-posedness in H^s, $s > 1/2$. This is a joint work with Anna Kazeykina (U. Paris-Sud).

A non-compactness result on the fractional Yamabe problem in large dimensions

Monica Musso

Pontificia Universidad Católica de Chile

Let (X^{n+1}, g^+) be an $(n+1)$-dimensional asymptotically hyperbolic manifold with a conformal infinity $(M^n, [h])$. The fractional Yamabe problem addresses to solve

$$P^\gamma[g^+, h](u) = cu^{\frac{n+2\gamma}{n-2\gamma}}, \quad u > 0 \quad \text{on } M$$

where $c \in \mathbb{R}$ and $P^\gamma[g^+, h]$ is the fractional conformal Laplacian whose principal symbol is $(-\Delta)^\gamma$. In this paper, we construct a metric on the half space $X = \mathbb{R}^{n+1}_+$, which is conformally equivalent to the unit ball, for which the solution set of the fractional Yamabe equation is non-compact provided that $n \geq 24$ for $\gamma \in (0, \gamma^*)$ and $n \geq 25$ for $\gamma \in [\gamma^*, 1)$ where $\gamma^* \in (0, 1)$ is a certain transition exponent. The value of γ^* turns out to be approximately 0.940197.

This is a joint work with Seunghyeok Kim and Juncheng Wei.

Large solutions for some local and non-local nonlinear elliptic equations

Alexander Quaas
Universidad Técnica Federico Santa María

We study existence, uniqueness and asymptotic behavior near the boundary of solutions of problems of the type

$$-I(u) + f(u) = 0 \quad \text{in}\, \Omega,$$

$$u = +\infty \qquad \text{on}\, \partial\Omega,$$

where Ω is a bounded smooth domain in $\mathbb{R}^N$, $N > 1$, I is a local or nonlocal elliptic (maybe degenerate) operator. The model for the function f is the power nonlinearity $f(s) = s^p$ with $p > 1$.

On a fractional mean curvature flow

Mariel Sáez
Pontificia Universidad Católica de Chile

In this talk I will discuss a fractional analog to the classical mean curvature flow. Namely, we consider the evolution of surfaces with normal speed equal to the fractional mean curvature and analyze their behavior under suitable assumptions. I will discuss in more depth the evolution of graphical hyper-surfaces, which is an important model in the local case.

This is joint work with Enrico Valdinoci

The positive solutions for the nonlinear Schrödinger systems with mixed couplings

Yohei Sato
Saitama University

We study the nonlinear Schrödinger systems which consist of three equations with variational structure. The functional for our systems has three coupling terms. In particular, we consider the case where the coefficient of two coupling terms are negative and the coefficient of one coupling term is positive. I would like to talk about the existence of positive solutions and the shape of the solutions when positive coefficient are sufficiently large. This is joint work with Prof. Z.-Q. Wang and Prof. J. Byeon.

Blowup in infinite time for competitive system of chemotaxis

Takashi Suzuki
Osaka University

We study competitive system of chemotaxis in two space dimension, proposed in the context of cell sorting and tumor micro-environment. If the chemotactic mobilities of the species are comparable, blowup in infinite time occurs only when the initial total masses are "quantized". Related and open problems are also presented.

Blowup behavior for a degenerate elliptic sinh-Poisson equation with variable intensities

Ryo Takahashi
Nara University of Education

In this talk, we provide a blow-up structure for solution sequences to an elliptic sinh-Poisson equation with variable intensities arising in the context of the statistical mechanics description of two-dimensional turbulence. The vortex intensities are described in terms of a probability measure $\mathcal{P}$ defined on the interval $[-1, 1]$. Under the Dirichlet boundary condition, we establish the exclusion of boundary blowup points, we show that the concentration mass does not have the residual L^1-terms and we determine the location of blowup points in terms of Kirchhoff's Hamiltonian. We allow $\mathcal{P}$ to be a general Borel measure, which could be "degenerate" in the sense that $\mathcal{P}(\{\alpha_-^*\}) = 0 = \mathcal{P}(\{\alpha_+^*\})$, where $\alpha_-^* = \min \operatorname{supp} \mathcal{P}$ and $\alpha_+^* = \max \operatorname{supp} \mathcal{P}$. Our main results are new for the standard sinh-Poisson equation as well.

Multiple-angle formula of generalized trigonometric functions with two parameters

Shingo Takeuchi
Shibaura Institute of Technology

Generalized trigonometric functions with two parameters were introduced by Drábek and Manásevich in 1999 to study an inhomogeneous eigenvalue problem of p-Laplacian. Concerning these functions, no multiple-angle formula has been known except for the classical case and a special case discovered by Edmunds-Gurka-Lang in 2012, not to mention addition theorems. In this talk, we will present a new multiple-angle formula which is established between two kinds of generalized trigonometric functions, and apply the formula to some problems for p-Laplacian.

A new deformation method for singular perturbation problems and applications

Kazunaga Tanaka
Waseda University

In this talk we propose a new deformation argument for singular perturbation problems, which is related to Pohozaev identity and inspired by joint works with Byeon [1,2] and Hirata and Ikoma [3]. We also give some applications to a nonlinear elliptic problems in a perturbed cylindrical domains etc.

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A symmetry-breaking bifurcation for a one-dimensional Liouville type equation

Satoshi Tanaka

Okayama University of Science

In this talk we consider the two-point boundary value problem for the one-dimensional Liouville type equation

$$\begin{cases} u'' + \lambda |x|^l e^u = 0, & x \in (-1,1), \\ u(-1) = u(1) = 0, \end{cases} \tag{1}$$

where $\lambda > 0$ and $l > 0$.

Jacobsen and Schmitt [5] gave the exact multiplicity result of radial solutions for the multi-dimensional problem

$$\begin{cases} \Delta u + \lambda |x|^l e^u = 0 & \text{in } B, \\ u = 0 & \text{on } \partial B, \end{cases} \tag{2}$$

where $\lambda > 0$, $l \geq 0$ and $B := \{x \in \mathbf{R}^N : |x| < 1\}$. In the case $N = 1$, problem (2) is reduced to (1). We note here that every solution of (2) is positive in B, by the strong maximum principle. Jacobsen and Schmitt [5] proved the following (i)–(iii):

1. if $1 \leq N \leq 2$, then there exists $\lambda_* > 0$ such that (2) has exactly two radial solutions for $0 < \lambda < \lambda_*$, a unique radial solution for $\lambda = \lambda_*$ and no radial solution for $\lambda > \lambda_*$;
2. if $3 \leq N < 10 + 4l$, then (2) has infinitely many radial solutions when $\lambda = (l+2)(N-2)$ and a finite but large number of radial solutions when $|\lambda - (l+2)(N-2)|$ is sufficiently small;
3. if $N \geq 10 + 4l$, then (2) has a unique radial solution for $0 < \lambda < (l+2)(N-2)$ and no radial solution for $\lambda \geq (l+2)(N-2)$.

This result was established by Liouville [10], Gel'fand [3], Joseph and Lundgren [6] for problem (2) with $l = 0$, that is,

$$\begin{cases} \Delta u + \lambda e^u = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{3}$$

when $\Omega = B$. Gidas, Ni and Nirenberg's theorem [4] shows that every positive solution of (3) is radially symmetric when $\Omega = B$. However, when Ω is an annulus $A := \{x \in \mathbf{R}^N : a < |x| < b\}$, $a > 0$, problem (3) may has non-radial solutions. Indeed, Lin [9] showed that (3) has infinitely many symmetry-breaking bifurcation points when $N = 2$ and $\Omega = A$. Nagasaki and Suzuki [12] found that large non-radial solutions of (3) when $N = 2$ and $\Omega = A$. More precisely, for each sufficiently large $\mu > 0$, there exist $\lambda > 0$ and a non-radial solution u of (3) such that $\int_A e^u dx = \mu$ when $N = 2$ and $\Omega = A$. Dancer [2] proved that non-radial solution branches emanating from the symmetry-breaking bifurcation points found by Lin [9] are unbounded. More general potential and domain were considered by del Pino, Kowalczyk and Musso [1], and they constructed concentrating solutions. Recently, Miyamoto [11] proved the following result for problem (2).

Theorem A ([11]). *Let n_0 be the largest integer that is smaller than $1+\frac{l}{2}$ and let $\alpha_n := 2\log\frac{2l+4}{l+2-2n}$. All the radial solutions of* (2) *with $N=2$ can be written explicitly as*

$$\lambda(\alpha) = 2(l+2)^2(e^{-\alpha/2} - e^{-\alpha}), \quad U(r;\alpha) = \alpha - \log(1 + (e^{\alpha/2} - 1)r^{l+2}).$$

The radial solutions can be parameterized by the L^∞-norm, it has one turning point at $\lambda = \lambda(\alpha_0) = (l+2)/2$, and it blows up as $\lambda \downarrow 0$. For each $n \in \{1, 2, \cdots, n_0\}$, $(\lambda(\alpha_n), U(r;\alpha_n))$ is a symmetry breaking bifurcation point from which an unbounded branch consisting of non-radial solutions of (2) *with $N=2$ emanates, and $U(r;\alpha)$ is nondegenerate if $\alpha \neq \alpha_n$, $n = 0, 1, \cdots, n_0$. Each non-radial branch is in $(0, \lambda(\alpha_0))) \times \{u > 0\} \subset R \times H_0^2(B)$.*

When $N = 2$, radial solutions of problems (2) and (3) can be written explicitly, and hence, Lin [9] and Miyamoto [11] succeeded to find the bifurcation points. That is difficult even if we know exact solutions, much more difficult if we do not know it usually. When $N \neq 2$, we do not know exact radial solutions of (2) with $l > 0$. However, when $N = 1$, the structure of eigenvalues and eigenfunctions is well-known, and combing this fact with the comparison function introduced in [13], we can find the Morse indexes of even solutions of (1). Then we obtain a symmetry-breaking bifurcation point of (1) as follows.

Theorem 1 *Let $l > 0$. Then, for each $\alpha > 0$, there exists a unique $(\lambda(\alpha), U(x,\alpha))$ such that* (1) *with $\lambda = \lambda(\alpha)$ has a unique positive even solution $U = U(x,\alpha)$ such that $||U||_\infty = \alpha$. Moreover, there exist α_1 and α_2 such that $\alpha_* < \alpha_1 \leq \alpha_2$ and the following* (i)–(iv) *hold*:

1. *if $0 < \alpha < \alpha_*$, then $m(U) = 0$;*

2. *if $\alpha = \alpha_*$, then $m(U) = 0$ and $U(x,\alpha)$ is degenerate;*

3. *if $\alpha = \alpha_1$, then $m(U) = 1$, $U(x,\alpha_1)$ is degenerate and $(U, \lambda(\alpha_1))$ is a non-even bifurcation point;*

4. *if $\alpha > \alpha_2$, then $m(U) = 2$.*

Here, $m(U)$ is the Morse index of U to a solution of (1), that is, the number of negative eigenvalues μ of

$$\begin{cases} \phi'' + \lambda|x|^l e^{U(x)}\phi + \mu\phi = 0, & x \in (-1,1), \\ \phi(-1) = \phi(1) = 0. \end{cases} \tag{4}$$

A solution U of (1) is said to be degenerate if $\mu = 0$ is an eigenvalue of (4). Otherwise, it is said to be nondegenerate.

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Some properties of zero-th order nonlocal operators approaching the fractional Laplacian

Erwin Topp Paredes
Universidad Técnica Federico Santa María

In this talk I am going to present several results concerning zero-th order nonlocal Dirichlet problems with the form

$$\begin{cases} \mathcal{I}_\epsilon(u) &= f \quad \text{in } \Omega \\ u &= 0 \quad \text{in } \Omega^c, \end{cases} \tag{P}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain, $f \in C(\bar{\Omega})$, and for $\sigma \in (0,1)$ given, the operator $\mathcal{I}_\epsilon$ has the form

$$\mathcal{I}_\epsilon(u,x) = \int_{\mathbb{R}^N} \frac{(u(x)-u(y))dy}{\epsilon^{N+2\sigma} + |x-y|^{N+2\sigma}},$$

which is an operator approaching the fractional Laplacian of order 2σ as $\epsilon \to 0$ (in the viscosity sense).

We present results concerning compactness (in $\bar{\Omega}$) for the family of solutions $(u_\epsilon)_\epsilon$ of (P), well-posedness results for the limit problem under rather weak assumptions on the regularity of $\partial\Omega$, and a study of the evolution of the interior regularity of the family as $\epsilon \to 0$.

Joint works with Patricio Felmer (DIM, Universidad de Chile) and Disson Dos Prazeres (Universidade Federal de Sergipe-UFS).