K_1-valued Alexander polynomials of knots

野坂 武史 (Takefumi Nosaka)

東京工業大学 (Tokyo Institute of Technology)

Motivation :

Motivation 1 : Unify many Alexander polynomials in non commutative coeffcients

Motivation 2 : Application from the K_1-groups.

Motivation 1 : Unify many Alexander polynomials in non commutative coeffcients

Related topics

- Higher order Alex. poly [Cochran]
- Solvable concordant filter[C-Orr-Teich
- Loc indicable pre. grp [Harvey]
- twisted Alex. polynomial [Lin, Wada]

※ [Milnor] Whitehead torsion from K_1-groups.

Motivation 2 : Application from the K_1-groups.

Motivation 1 : Unify many Alexander polynomials in non commutative coeffcients

Related topics

- Higher order Alex. poly [Cochran]
- Solvable concordant filter[C-Orr-Teich
- Loc indicable pre. grp [Harvey]
- twisted Alex. polynomial [Lin, Wada]

※ [Milnor] Whitehead torsion from K_1-groups.

Motivation 2 : Application from the K_1-groups.

Topic

- Invariants of $\pi_1(S^3 \setminus K) \to G$, where $|G| < \infty$.
- Settings for Casson-Gordon invariants

REM. Metabelian twited Alexander poly [Kirk-Livingston]

Results

Results

Thm. 1 (N.)

For $^{\forall}$ grp. hom $\pi_1(S^3 \setminus L) \to G$ & fixed meridian $\mathfrak{m}$ we defined K_1-valued (1-vari) Alexander polynomials.

Results

Thm. 1 (N.)

For $^{\forall}$ grp. hom $\pi_1(S^3 \setminus L) \to G$ & fixed meridian $\mathfrak{m}$
we defined K_1-valued (1-vari) Alexander polynomials.

Thm 2 (N) If G is "meta-finite",

$^{\exists}$ a symmetry of the K_1-valued Alexander polynomial.

cf. $\Delta_K(t) \doteq \Delta_K(t^{-1})$ of twisted Alexander poly.

Results

Thm. 1 (N.)

For $^{\forall}$ grp. hom $\pi_1(S^3 \setminus L) \to G$ & fixed meridian $\mathfrak{m}$
we defined K_1-valued (1-vari) Alexander polynomials.

Thm 2 (N) If G is "meta-finite",

$^{\exists}$ a symmetry of the K_1-valued Alexander polynomial.

cf. $\Delta_K(t) \stackrel{\bullet}{=} \Delta_K(t^{-1})$ of twisted Alexander poly.

Ex. Computation of K_1-valued Alexander poly.
for Torus knots, 2-bridge knots of genus 1

Today's contents

Today's contents

Review : Alexander poly from knot diagrams

Review : Alexander poly from knot diagrams

Wirtinger presentation $\langle x_1, \ldots, x_n | r_1, \ldots, r_{n-1} \rangle$

$\mathrm{Ab} : \pi_1(S^3 \setminus K) \longrightarrow \mathbb{Z} = \langle t \rangle$

$\Longrightarrow$

$\alpha : \mathbb{Z}[\pi_1(S^3 \setminus K)] \longrightarrow \mathbb{Z}[t^{\pm 1}]$ **ring hom.**

Review : Alexander poly from knot diagrams

Wirtinger presentation $\langle x_1, \ldots, x_n | r_1, \ldots, r_{n-1} \rangle$

Ab : $\pi_1(S^3 \setminus K) \longrightarrow \mathbb{Z} = \langle t \rangle$

$\Longrightarrow$

$\alpha : \mathbb{Z}[\pi_1(S^3 \setminus K)] \longrightarrow \mathbb{Z}[t^{\pm 1}]$ **ring hom.**

Let's consider the $n \times (n-1)$**-matrix over** $\mathbb{Z}[t^{\pm 1}]$ **:**

$$\Phi_{\alpha,k} := \begin{pmatrix} \alpha(\frac{\partial r_1}{\partial x_1}) & \cdots & \cdots & \alpha(\frac{\partial r_1}{\partial x_n}) \\ \vdots & & \ddots & \vdots \\ \alpha(\frac{\partial r_{n-1}}{\partial x_1}) & \cdots & \cdots & \alpha(\frac{\partial r_{n-1}}{\partial x_n}) \end{pmatrix}$$

Review : Alexander poly from knot diagrams

Wirtinger presentation $\langle x_1, \ldots, x_n | r_1, \ldots, r_{n-1} \rangle$

$\mathrm{Ab} : \pi_1(S^3 \setminus K) \longrightarrow \mathbb{Z} = \langle t \rangle$

$\Longrightarrow$

$\alpha : \mathbb{Z}[\pi_1(S^3 \setminus K)] \longrightarrow \mathbb{Z}[t^{\pm 1}]$ ring hom.

Let's consider the $(n-1) \times (n-1)$-matrix over $\mathbb{Z}[t^{\pm 1}]$:

$$\Phi_{\alpha,k} := \begin{pmatrix} \alpha(\frac{\partial r_1}{\partial x_1}) & \cdots & \cdots & \alpha(\frac{\partial r_1}{\partial x_n}) \\ \vdots & & \ddots & \vdots \\ \alpha(\frac{\partial r_{n-1}}{\partial x_1}) & \cdots & \cdots & \alpha(\frac{\partial r_{n-1}}{\partial x_n}) \end{pmatrix}$$

Remove the k-th row.

Review : Alexander poly from knot diagrams

Wirtinger presentation $\langle x_1, \dots, x_n | r_1, \dots, r_{n-1} \rangle$

$\mathrm{Ab} : \pi_1(S^3 \setminus K) \longrightarrow \mathbb{Z} = \langle t \rangle$

$\Longrightarrow$

$\alpha : \mathbb{Z}[\pi_1(S^3 \setminus K)] \longrightarrow \mathbb{Z}[t^{\pm 1}]$ ring hom.

Let's consider the $(n-1) \times (n-1)$-matrix over $\mathbb{Z}[t^{\pm 1}]$:

$$\Phi_{\alpha,k} := \begin{pmatrix} \alpha(\frac{\partial r_1}{\partial x_1}) & \cdots & \cdots & \alpha(\frac{\partial r_1}{\partial x_n}) \\ \vdots & & \ddots & \vdots \\ \alpha(\frac{\partial r_{n-1}}{\partial x_1}) & \cdots & \cdots & \alpha(\frac{\partial r_{n-1}}{\partial x_n}) \end{pmatrix}$$

Remove the k-th row.

Alexander poly. $\Delta_K(t) \stackrel{\bullet}{:=} \det(\Phi_{\alpha,k})$.

Review : Alexander poly from knot diagrams

Wirtinger presentation $\langle x_1, \ldots, x_n | r_1, \ldots, r_{n-1} \rangle$

Ab $: \pi_1(S^3 \setminus K) \longrightarrow \mathbb{Z} = \langle t \rangle$

$\Longrightarrow$

$\alpha : \mathbb{Z}[\pi_1(S^3 \setminus K)] \longrightarrow \mathbb{Z}[t^{\pm 1}]$ **ring hom.**

Two Idea to non-commutativity

Wirtinger pre. $\langle x_1, \ldots, x_n | r_1, \ldots, r_{n-1} \rangle$

$$f : \pi_1(S^3 \setminus K) \longrightarrow G$$

$$\Longrightarrow$$

$$\alpha : \mathbb{Z}[\pi_1(S^3 \setminus K)] \longrightarrow R \qquad \text{ring hom.}$$

Two Idea to non-commutativity

Wirtinger pre. $\langle x_1, \ldots, x_n | r_1, \ldots, r_{n-1} \rangle$

$f : \pi_1(S^3 \setminus K) \longrightarrow G$

$\Longrightarrow$

$\alpha : \mathbb{Z}[\pi_1(S^3 \setminus K)] \longrightarrow R$ **ring hom.**

Idea 1 Change to a good non-comm ring R !

A. Let's use Novikov rings !

Idea 2

Two Idea to non-commutativity

Wirtinger pre. $\langle x_1, \ldots, x_n | r_1, \ldots, r_{n-1} \rangle$

$f : \pi_1(S^3 \setminus K) \longrightarrow G$

$\Longrightarrow$

$\alpha : \mathbb{Z}[\pi_1(S^3 \setminus K)] \longrightarrow R$ ring hom.

Idea 1 Change to a good non-comm ring R !

A. Let's use Novikov rings !

Idea 2

Regard not only determinants, but also matrices as invariants.

A. Use K_1-group !

Idea 1

Change $\mathbb{Z}[t^{\pm 1}]$ to a Novikov ring $\mathcal{A}_\kappa[\![\tau]\!]$.

Idea 1

Change $\mathbb{Z}[t^{\pm 1}]$ to a Novikov ring $\mathcal{A}_\kappa[\![\tau]\!]$.

$\mathcal{A}$:a non-comm. ring $\qquad \kappa : \mathcal{A} \to \mathcal{A}$ ring iso.

$$\mathcal{A}_\kappa[\![\tau]\!] := \{a_N\tau^N + a_{N+1}\tau^{N+1} + \cdots \mid a_i \in \mathcal{A}, \quad N \in \mathbb{Z}\}$$

Here, the relation $a\tau^m = \tau^m\kappa^m(a)$ holds.

Idea 1

Change $\mathbb{Z}[t^{\pm 1}]$ to a Novikov ring $\mathcal{A}_\kappa[\![\tau]\!]$.

$\mathcal{A}$:a non-comm. ring $\qquad \kappa : \mathcal{A} \to \mathcal{A}$ ring iso.

$$\mathcal{A}_\kappa[\![\tau]\!] := \{a_N\tau^N + a_{N+1}\tau^{N+1} + \cdots \mid a_i \in \mathcal{A}, \quad N \in \mathbb{Z}\}$$

Here, the relation $a\tau^m = \tau^m\kappa^m(a)$ holds.

<u>Merits of Novikov rings</u>

- Invertibility of $1-\tau$: $(1-\tau)^{-1} = 1 + \tau + \tau^2 + \cdots$.
- Many matrices on Novikov rings are invertible.
- We can define Logarithm.
- Relations to S^1-valued Morse theory, fibered-knots.

Idea 1

Change $\mathbb{Z}[t^{\pm 1}]$ to a Novikov ring $\mathcal{A}_{\kappa}[\![\tau]\!]$.

Idea 1

Change $\mathbb{Z}[t^{\pm 1}]$ to a Novikov ring $\mathcal{A}_\kappa[\![\tau]\!]$.

Today's assumption

Fix a ring hom. $\rho : \mathbb{Z}[\pi_1(S^3 \setminus K)] \longrightarrow \mathcal{A}_\kappa[\![\tau]\!]$, and $\exists$ meridian $\mathfrak{m}$ s.t. $\rho(\mathfrak{m}) = \tau$.

Idea 1

Change $\mathbb{Z}[t^{\pm 1}]$ to a Novikov ring $\mathcal{A}_\kappa[\![\tau]\!]$.

Today's assumption

Fix a ring hom. $\rho : \mathbb{Z}[\pi_1(S^3 \setminus K)] \longrightarrow \mathcal{A}_\kappa[\![\tau]\!]$, and
$\exists$ meridian $\mathfrak{m}$ s.t. $\rho(\mathfrak{m}) = \tau$.

Example 1

For any grp. hom $\pi_1(S^3 \setminus K) \longrightarrow H \rtimes \mathbb{Z}$
let $\mathcal{A}$ be $\mathbb{Q}[H]$, and
κ be the isomorphism $H \xrightarrow{\sim} H$ from $H \rtimes \mathbb{Z}$.

Idea 1

Change $\mathbb{Z}[t^{\pm 1}]$ to a Novikov ring $\mathcal{A}_\kappa[\![\tau]\!]$.

Today's assumption

Fix a ring hom. $\rho : \mathbb{Z}[\pi_1(S^3 \setminus K)] \longrightarrow \mathcal{A}_\kappa[\![\tau]\!]$, and $\exists$ meridian $\mathfrak{m}$ s.t. $\rho(\mathfrak{m}) = \tau$.

Example 1

For any grp. hom $\pi_1(S^3 \setminus K) \longrightarrow H \rtimes \mathbb{Z}$

let $\mathcal{A}$ be $\mathbb{Q}[H]$, and

κ be the isomorphism $H \xrightarrow{\sim} H$ from $H \rtimes \mathbb{Z}$.

Example 2

$\forall$grp. hom $\rho : \pi_1(S^3 \setminus K) \longrightarrow G$. $\quad H := \mathrm{Im}(\rho([\pi_K, \pi_K]))$,

$\Longrightarrow$ We have a situation in Example 1 .

Idea 2

Regard matricies in K_1-grp as topological invariants !

Idea 2

Regard matricies in K_1-grp as topological invariants !

R:ring with unit $GL_n(R)$: general marices group

$$GL_1(R) \hookrightarrow GL_2(R) \hookrightarrow GL_3(R) \hookrightarrow \cdots \hookrightarrow GL(R)$$

K_1-group, $K_1(R)$, is the abelianization $GL(R)_{\mathrm{ab}}$.

Idea 2

Regard matricies in K_1-grp as topological invariants !

R:ring with unit $\qquad$ $GL_n(R)$: general marices group

$$GL_1(R) \hookrightarrow GL_2(R) \hookrightarrow GL_3(R) \hookrightarrow \cdots \hookrightarrow GL(R)$$

K_1-group, $K_1(R)$, is the abelianization $GL(R)_{\mathrm{ab}}$.

Whitehead Lemma

$[GL(R), GL(R)]$ equals
the subgroup generated by elementary matrices.

Idea 2

Regard matricies in K_1-grp as topological invariants !

R:ring with unit $\qquad GL_n(R)$: general marices group

$$GL_1(R) \hookrightarrow GL_2(R) \hookrightarrow GL_3(R) \hookrightarrow \cdots \hookrightarrow GL(R)$$

K_1-group, $K_1(R)$, is the abelianization $GL(R)_{\mathrm{ab}}$.

Whitehead Lemma

$[GL(R), GL(R)]$ equals
the subgroup generated by elementary matrices.

$\Longrightarrow$ K_1-grp is a universal obj. of all matrices
invariant w.r.t. elementary transformations.

Example R a field $\Longrightarrow$ det:$K_1(R) \cong R^{\times}$.

Today's contents

Today's contents

K_1-Alexander poly from a knot diagrma

K_1-Alexander poly from a knot diagrma

Wirtinger pre $\langle x_1, \ldots, x_n | r_1, \ldots, r_{n-1} \rangle$

$\rho : \mathbb{Z}[\pi_1(S^3 \setminus K)] \longrightarrow \mathcal{A}[\tau]\!]$ as above.

$\Longrightarrow$

K_1-Alexander poly from a knot diagrma

Wirtinger pre $\langle x_1, \ldots, x_n | r_1, \ldots, r_{n-1} \rangle$

$\rho : \mathbb{Z}[\pi_1(S^3 \setminus K)] \longrightarrow \mathcal{A}[\![\tau]\!]$ as above.

$\Longrightarrow$

Consider the $(n-1) \times (n-1)$-matrix over $\mathcal{A}[\![\tau]\!]$:

$$\Phi_{\rho,k} := \begin{pmatrix} \rho(\frac{\partial r_1}{\partial x_1}) & \cdots & \cdots & \rho(\frac{\partial r_1}{\partial x_n}) \\ \vdots & & \ddots & \vdots \\ \rho(\frac{\partial r_{n-1}}{\partial x_n}) & \cdots & \cdots & \rho(\frac{\partial r_{n-1}}{\partial x_n}) \end{pmatrix}$$

K_1-Alexander poly from a knot diagrma

Wirtinger pre $\langle x_1, \ldots, x_n | r_1, \ldots, r_{n-1} \rangle$

$\rho : \mathbb{Z}[\pi_1(S^3 \setminus K)] \longrightarrow \mathcal{A}[\![\tau]\!]$ as above.

$\Longrightarrow$

Consider the $(n-1) \times (n-1)$-matrix over $\mathcal{A}[\![\tau]\!]$:

$$\Phi_{\rho,k} := \begin{pmatrix} \rho(\frac{\partial r_1}{\partial x_1}) & \cdots \quad \cdots & \rho(\frac{\partial r_1}{\partial x_n}) \\ \vdots & \ddots & \vdots \\ \rho(\frac{\partial r_{n-1}}{\partial x_n}) & \cdots \quad \cdots & \rho(\frac{\partial r_{n-1}}{\partial x_n}) \end{pmatrix}$$

Thm.(N.) (cf. [Wada, Kitayama])

Suppose $[\Phi_{\rho,k}]$ is invertible. K_1-Alexander polynomial is

$\Delta_\rho := [\Phi_{\rho,k}] \in K_1(\mathcal{A}[\![\tau]\!])/\{\pm\tau^m\}_{m\in\mathbb{Z}}$

It depends only on the choice of ρ & $\mathfrak{m}$.

Q. Is the matrix $\Phi_{\rho,k}$ always invertible ?

Q. **Is the matrix $\Phi_{\rho,k}$ always invertible ?**

Prop. **The followings are equivalent.**

- K is a fibered knot.
- For $^{\forall}\rho$, $\Phi_{\rho,k}$ is invertible.

※ For some non-fibered kntos, $^{\exists}\rho$ s.t. $\Phi_{\rho,k}$ is invertible.

※ Key is a work of [Friedl] on the relation to acyclicity of Novikov homology.

Example of computing K_1-Alexander poly.

Example of computing K_1-Alexander poly.

Torus knot $T_{p,q}$ $\qquad \pi_1 = \langle x, y | x^p = y^q \rangle$

Prop.

$$\Delta_\rho = \frac{(\rho(x^p) - 1)(\rho(\mathfrak{m}) - 1)}{(\rho(x) - 1)(\rho(y) - 1)}$$

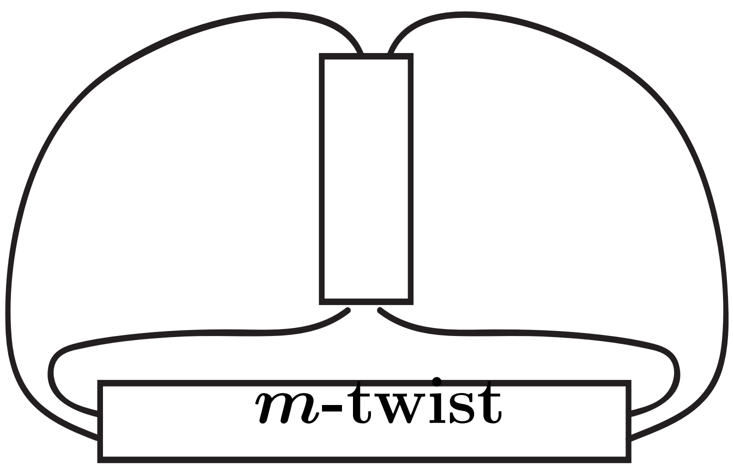
Example of computing K_1-Alexander poly.

Torus knot $T_{p,q}$ $\qquad$ $\pi_1 = \langle x, y \mid x^p = y^q \rangle$

Prop.

$$\Delta_\rho = \frac{(\rho(x^p) - 1)(\rho(\mathfrak{m}) - 1)}{(\rho(x) - 1)(\rho(y) - 1)}$$

2-bridge knot of genus 1



Example of computing K_1-Alexander poly.

Torus knot $T_{p,q}$ $\qquad \pi_1 = \langle x, y | x^p = y^q \rangle$

Prop.

$$\Delta_\rho = \frac{(\rho(x^p) - 1)(\rho(\mathfrak{m}) - 1)}{(\rho(x) - 1)(\rho(y) - 1)}$$

2-bridge knot of genus 1

Example of computing K_1-Alexander poly.

Torus knot $T_{p,q}$ $\qquad \pi_1 = \langle x, y | x^p = y^q \rangle$

Prop.

$$\Delta_\rho = \frac{(\rho(x^p) - 1)(\rho(\mathfrak{m}) - 1)}{(\rho(x) - 1)(\rho(y) - 1)}$$

2-bridge knot of genus 1

$\pi_1(S^3 \backslash L) \cong \langle x, y | w^n x = y w^n \rangle$, where $w = (xy^{-1})^m (x^{-1} y)^m$,

Prop. Let $n > 0$. Δ_ρ equals

$$\rho(w^n) + (1 - \rho(y)) \frac{1 - \rho(w^n)}{1 - \rho(w)} (1 - \rho(w x^{-1})) \frac{1 - \rho(xy^{-1})^m}{1 - \rho(xy^{-1})}$$

Examples of non-triviality for $3_1, 4_1, 5_2$.

Today's contents

Today's contents

Assumption in this §

ρ is obtained from $\pi_1(S^3 \setminus K) \xrightarrow{\text{grp.hom}} H \rtimes \mathbb{Z}$
where, H is a finite group.

Assumption in this §

ρ is obtained from $\pi_1(S^3 \setminus K) \xrightarrow{\text{grp.hom}} H \rtimes \mathbb{Z}$

where, H is a finite group.

<u>Rem</u> $\exists m \in \mathbb{N}$ s.t. $\kappa^m = \mathrm{id}_H$.

Thus, $\tau^m a = a\tau^m$, i.e., τ^m is commutative in $\mathcal{A}_\kappa[\![\tau]\!]$.

A ring hom. & Morita equivalence

I defined a ring hom. $\Upsilon : \mathcal{A}_\kappa[\![\tau]\!] \to \mathrm{Mat}(m \times m; \mathcal{A}_\kappa[\![\tau^m]\!])$.

$\Longrightarrow$

$$K_1(\mathcal{A}_\kappa[\![\tau]\!]) \xrightarrow{\Upsilon_*} K_1(\mathrm{Mat}(m \times m; \mathcal{A}_\kappa[\![\tau^m]\!])) \cong K_1(\mathcal{A}_\kappa[\![\tau^m]\!])$$

Assumption in this §

ρ is obtained from $\pi_1(S^3 \setminus K) \xrightarrow{\text{grp.hom}} H \rtimes \mathbb{Z}$
where, H is a finite group.

<u>Rem</u> $\exists m \in \mathbb{N}$ s.t. $\kappa^m = \mathrm{id}_H$.

Thus, $\tau^m a = a\tau^m$, i.e., τ^m is commutative in $\mathcal{A}_\kappa[\![\tau]\!]$.

A ring hom. & Morita equivalence

I defined a ring hom. $\Upsilon : \mathcal{A}_\kappa[\![\tau]\!] \to \mathrm{Mat}(m \times m; \mathcal{A}_\kappa[\![\tau^m]\!])$.

$\Longrightarrow$

$$K_1(\mathcal{A}_\kappa[\![\tau]\!]) \xrightarrow{\Upsilon_*} K_1(\mathrm{Mat}(m \times m; \mathcal{A}_\kappa[\![\tau^m]\!])) \cong K_1(\mathcal{A}_\kappa[\![\tau^m]\!])$$

Let's study the pushforward $\Upsilon_*(\Delta_\rho)$!

$p : E_K^m \to S^3 \setminus K$ be the m-fold cyclic covering.

$p : E_K^m \to S^3 \setminus K$ be the m-fold cyclic covering.

Thm. (N.)

$\Upsilon_*(\Delta_\rho)$= Reidemeister torsion of E_K^m w.r.t. $\rho \circ p$.

Thm. (Symmetry)[N.]

If (1) $\rho(\mathfrak{l}) = 0$ or (2) $\Upsilon(1 - \rho(\mathfrak{l}))$ is invertible,

$\Longrightarrow$

$$\overline{\Upsilon_*(\Delta_\rho)} = \Upsilon_*(\Delta_\rho) \in \frac{K_1(\mathcal{A}(\tau))}{\{K_1(\mathcal{A}), \tau^m\}}$$

$p : E_K^m \to S^3 \setminus K$ be the m-fold cyclic covering.

Thm. (N.)

$\Upsilon_*(\Delta_\rho)$= Reidemeister torsion of E_K^m w.r.t.$\rho \circ p$.

Thm. (Symmetry)[N.]

If (1) $\rho(\mathfrak{l}) = 0$ or (2) $\Upsilon(1 - \rho(\mathfrak{l}))$ is invertible,

$\Longrightarrow$

$$\overline{\Upsilon_*(\Delta_\rho)} = \Upsilon_*(\Delta_\rho) \in \frac{K_1(\mathcal{A}(\tau))}{\{K_1(\mathcal{A}), \tau^m\}}$$

$p : E_K^m \to S^3 \setminus K$ be the m-fold cyclic covering.

Thm. (N.)

$\Upsilon_*(\Delta_\rho)$= Reidemeister torsion of E_K^m w.r.t.$\rho \circ p$.

Thm. (Symmetry)[N.]

If (1) $\rho(\mathfrak{l}) = 0$ or (2) $\Upsilon(1 - \rho(\mathfrak{l}))$ is invertible,

$\Longrightarrow$

$$\overline{\Upsilon_*(\Delta_\rho)} = \Upsilon_*(\Delta_\rho) \in \frac{K_1(\mathcal{A}(\tau))}{\{K_1(\mathcal{A}), \tau^m\}}$$

Thm. (A slight generalization of [Kirk-Livingson])[N.]

K: a topological slice knot.

ρ: a meta-abelian hom in [Casson-Gordon]

$\Longrightarrow$

${}^\exists f(t), \quad \Upsilon_*(\Delta_\rho) = (1-t)f(t)f(t^{-1}) \in$ a ring

In summary

In summary

In summary

- Def. of K_1-Alexander poly w.r.t. $\pi_1(S^3 \setminus K) \to G$.
  - Knot diagrams & Fox derivative
  - Heegaard decomp. & Fox derivative
  - Reidemeister torsion
  - S^1-valued Morse theory if $\rho(\mathfrak{l}) = 0$

In summary

- Def. of K_1-Alexander poly w.r.t. $\pi_1(S^3 \setminus K) \to G$.
  - Knot diagrams & Fox derivative
  - Heegaard decomp. & Fox derivative
  - Reidemeister torsion
  - S^1-valued Morse theory if $\rho(\mathfrak{l}) = 0$

- If G is finite or meta-finite,

the K_1-Alexander polynomial has a symmetry,

is related to some Reidemeister torsion.

In summary

- Def. of K_1-Alexander poly w.r.t. $\pi_1(S^3 \setminus K) \to G$.
  - Knot diagrams & Fox derivative
  - Heegaard decomp. & Fox derivative
  - Reidemeister torsion
  - S^1-valued Morse theory if $\rho(\mathfrak{l}) = 0$

- If G is finite or meta-finite,

the K_1-Alexander polynomial has a symmetry,

is related to some Reidemeister torsion.

Thank you