CORRIGENDA TO "AN INVERSE PROBLEM FOR A CLASS OF CANONICAL SYSTEMS AND ITS APPLICATIONS TO SELF-RECIPROCAL POLYNOMIALS"

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Corrections

(1) p.278, l.1: remove "has no zeros on the real line and that", because the assumption unnecessary.

(2) p.285. l.1–6: revise these lines from "Then" to "integer n." as follows:

"Then, for the conjugate operator $\mathsf{P}_n^{**} := \mathsf{J}\mathsf{P}_n^*\mathsf{J} : V_a \to V_a$ of P_n^* by the involution $\mathsf{J} : V_a \mapsto V_a$ defined by

$$\mathsf{J}\phi = \sum_{k=-\infty}^{\infty} v(k)X(k) + \sum_{l=-\infty}^{\infty} u(l)Y(l) \quad (\phi \in V_a),$$

we have

$$\mathsf{P}_n^{**}\phi = \mathsf{J}\mathsf{P}_n^*\mathsf{J}\phi = \sum_{k=-\infty}^{n-1} u(k)X(k) + \sum_{l=0}^{\infty} v(l)Y(l) \quad (\phi \in V_a).$$

Therefore, $\mathsf{P}_n := \mathsf{P}_n^{**}\mathsf{P}_n^*$ maps V_a into $V_{a,n}$ for every nonnegative integer n."

(3) p.286, l.3; replacing the line with "and $\Theta\mathsf{J}\mathsf{P}_n(W_{a,n}) \subset W_{a,n}$ for each nonnegative integer n, where

$$W_{a,n} := V_{a,n} + \Theta\mathsf{J}\mathsf{P}_n V_{a,n}.\text{"}$$

(4) Replace $V_{a,n}$ with $W_{a,n}$ on p. 286, (2.9), l. 12, l.13; p. 297, l.-2; p. 298, Lemma 4.2 and its proof; p. 299, l.-1, l.-10; p. 305, Proposition 4.8; p. 313, l.4, l.10.

(5) p.297, Lemma 4.1: delete the second claim (2) and its proof, because it is not correct.

(6) p.297, Proof of Lemma 4.1: delete the first sentence, because it is not correct. Replace the second sentence "Therefore, it is ..." with "It is sufficient to prove that E is invertible on p_1V_a and p_2V_a, since $e^{\pm 2itz}(1/E(z))f(z) = g(z)$ is impossible for any $0 \neq f, g \in L^2(\mathbb{T}_q)$".

(7) p.298, Lemma 4.2: delete the assumption "that $E(z)$ of (1.3) has no zeros on the real line and" because it is unnecessary.

(8) p.298, Proof of Lemma 4.2: modify the explanation for the compactness of $\Theta\mathsf{J}\mathsf{P}_n$ in lines 1–6 of the proof as follows: "Let $n \geq 1$. By definition, P_n is a projection from V_a into $V_{a,n}$, so $\Theta\mathsf{J}\mathsf{P}_n$ is an operator on $W_{a,n}$. The image of $W_{a,n}$ by $\mathsf{E}^\sharp\mathsf{J}\mathsf{P}_n$ is finite dimensional by definition of $\mathsf{E}^\sharp$ and (2.4), thus $\Theta\mathsf{J}\mathsf{P}_n = \mathsf{E}^{-1}(\mathsf{E}^\sharp\mathsf{J}\mathsf{P}_n)$ is a finite rank operator on $W_{a,n}$. Hence it is compact." Also, replace "see the proof of Lemma 4.1(2)" with "since $e^{\pm 2itz}(1/E(z))f(z) = g(z)$ is impossible for any $0 \neq f, g \in L^2(\mathbb{T}_q)$", because Lemma 4.1(2) can no longer be referenced by (6) above.

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(9) p.299, l.13: removed the phrase "$E(z) \neq 0$ for $z \in \mathbb{R}$ and" because it is unnecessary as well as (7) above.
(10) p.299, l.−10; p.300, Lemma 4.3 and its proof; p.305, Proposition 4.8; p.313, l.3, l.9: replace $\mathsf{E} \pm \mathsf{E}^\sharp \mathsf{J} \mathsf{P}_n$ with $\mathsf{I} \pm \Theta \mathsf{J} \mathsf{P}_n$.
(11) p.312, the fourth line in the proof of Proposition 4.8: revise "in (4.1)" to "on the left-hand side of (4.1)".
(12) p.316, Proposition 4.7: change the assumption "$E(z)$ has no zeros on the real line" to " $\det D_n(\underline{C}) \neq 0$ for every $1 \leq n \leq 2g$", because the latter condition is natural as a condition that $\gamma(t)$ is well-defined (cf. (14) below).
(13) p.298, Proposition 4.18: delete the assumption "that $E(z)$ has no zeros on the real line and" because it is unnecessary as well as (7).
(14) pp.322–325, Proof of Theorem 1.2(1): the original proof is insufficient to explain that γ_n is defined for all $1 \leq n \leq 2g$ if E belongs to the class HB and there is a gap in the discussion to the case $\gamma_n = 0, \infty$ for some n. Therefore, we revise the proof as follows.
   (a) revise the first three lines of the proof as follows:
   "For $\underline{C} = (C_g, C_{g-1}, \cdots, C_{-g}) \in \mathbb{R}^* \times \mathbb{R}^{2g-1} \times \mathbb{R}^*$, the polynomial
   $$f(T) := T^g \sum_{m=-g}^{g} C_{-m}\, T^m \ \in \mathbb{R}[T]$$
   and $f^\sharp(T) := T^{2g} f(1/T)$ have a common root if and only if $\det D_{2g}(\underline{C})$ is zero (Lemmas 11.5.11 and 11.5.12 of Q. I. Rahman, G. Schmeisser, Analytic theory of polynomials, London Mathematical Society Monographs. New Series, 26, The Clarendon Press, Oxford University Press, Oxford, 2002). The former is equivalent that E and $E^\sharp$ have a common zero, since $E(z) = q^{igz} f(q^{-iz})$ and $E^\sharp(z) = E(-z) = q^{igz} f^\sharp(q^{-iz})$. If E belongs to the class HB, it has no real zeros and $|E(\bar{z})| < |E(z)|$ in $\mathbb{C}_+$ by definition of the class HB. Therefore E and $E^\sharp$ have no common zeros. Hence $\det D_{2g}(\underline{C}) \neq 0$, which implies that $\det D_n(\underline{C}) \neq 0$ for every $1 \leq n \leq 2g$. Hence $\gamma_n \neq 0, \infty$ for every $1 \leq n \leq 2g$ by Theorem 4.8 (1). Therefore, it is sufficient to prove that $E(z)$ is not a function of the class HB if $\gamma_n < 0$ for some $1 \leq n \leq 2g$. We proceed in three steps as follows."
   (b) In lines 2–3 of Step 1, modify "one of $\gamma_n < 0$ or $\gamma_n = 0$ or $\gamma_n^{-1} = \infty$ " to "$\gamma_{n_0+1} < 0$".
   (c) Revise the first six lines of Step 3 as follows, and delete the discussion for **Case (i)**: "For the number n_0 of Step 1, we prove that $E(z)$ is not a function of the class HB if $\gamma_{n_0+1} < 0$. Considering the argument in Step 2, we assume that $E(a, z)$ is a function of the class HB for every $1 < a \leq q^{(n_0+1)/2}$ in both cases and find a contradiction."
(15) p.340: add the paper "Q. I. Rahman, G. Schmeisser, Analytic theory of polynomials, London Mathematical Society Monographs. New Series, 26, The Clarendon Press, Oxford University Press, Oxford, 2002." to the bibliography for the revision of the proof of Theorem 1.2 (1)

Typos

(1) p.275, l.−11: modify "fixed regular point" to "fixed point".
(2) p.284, l.−11: modify "is and abstract" to "is an abstract".
(3) p.290, definition of $e^\sharp_{1,n}$, $e^\sharp_{2,n}$, $e^\sharp_{3,n}$: modify "$C_{-(g-1)} \ \cdot^{\cdot^{\cdot}} \ C_{(g-1)}$" to "$C_{g-1} \ \cdot^{\cdot^{\cdot}} \ C_{-(g-1)}$".
(4) p.312: modify $\sum_{k=-1}^{n-1}$ to $\sum_{k=1}^{n-1}$ in the fourth line of (4.26), and modify $\sum_{k=-1}^{n-1}$ to $\sum_{k=1}^{n-1}$ in the third line of (4.28).

(5) p.314, l.−10: modify "By (4.4)" to "By (4.3)".
(6) p.314, Proposition 4.16: modify "$A_n^*(n+1)\ \cdots\ A_n^*(2g+1)$ and $B_n^*(n+1)\ \cdots\ B_n^*(2g+1)$" to "$A_n^*(2g+1)\ \cdots\ A_n^*(n+1)$ and $B_n^*(2g+1)\ \cdots\ B_n^*(n+1)$", respectively.
(7) p.316, Proof of Proposition 4.17: modify $\Omega_g(0)$ to Ω_0 in the first line of the proof.
(8) p.318, Proposition 4.18 (4): modify $\lim_{a\nearrow q^{n/2}}(A_q(a,z), B_q(a,z))$ to $\lim_{a\nearrow q^{n/2}}(A(a,z), B(a,z))$.
(9) p.323, the fifth line of Step 2: modify "since $\gamma(a)\neq 0$" to "when $\gamma(a)\neq 0$".
(10) p.323, the 10th line of Step 2: modify "$1\le a_0\le q^{n_0/2}$" to "$1\le a_0\le q^{(n_0+1)/2}$".
(11) p.324: modify the product $\prod_{\substack{|\rho|<R\\ E(\rho)=0}}$ in two places to $\prod_{\substack{|\rho|<R\\ E(a_0,\rho)=0}}$.
(12) p.325, the second line of **Case (ii)**: modify $a_0=(q^{(n_0+1)/2}-q^{n_0/2})/2$ to $a_0=(q^{(n_0+1)/2}+q^{n_0/2})/2$.
(13) p.325, l.−9: modify $K(t_0,iy,iy)$ to $K(a_0,iy,iy)$.

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