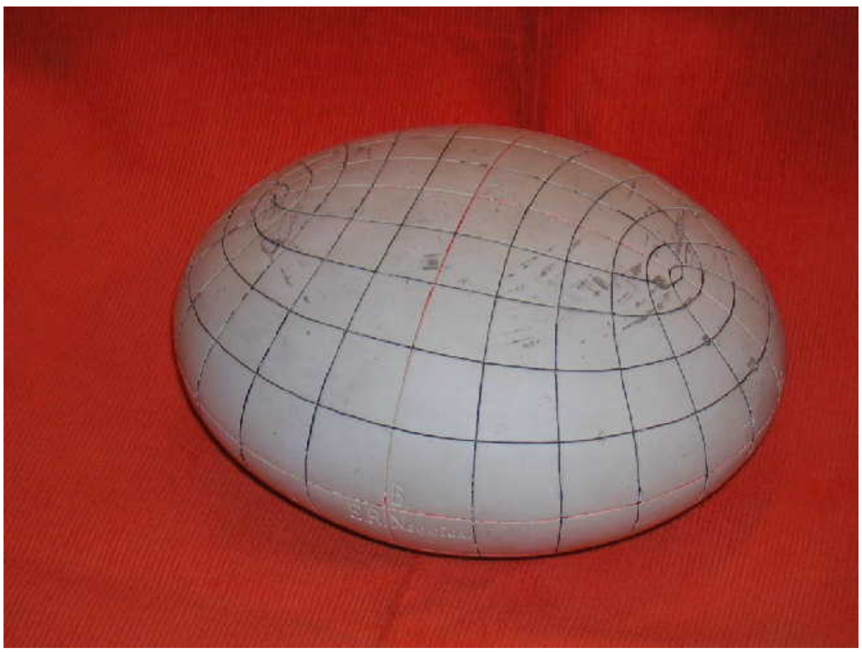
Umbilical points on surfaces, W Klingenberg, Durham University

In joint work with Brendan Guilfoyle we establish an upper bound for the winding number of the principal curvature foliation at any isolated umbilic of a surface in Euclidean three-space. Here is a model of the foliation by two families of mutually perpendicular curvature lines and two of the four umbilics on a triaxial ellipsoid. They have index 1/2.



In our talk we will give a non-technical introduction to the circle of contemporary analytic and geometric methods that enter in studying this problem of classical differential geometry. Here is a brief outline of our work.

1. **Space of oriented lines** Our approach uses the represention of a classical surface S in $\mathbb{R}^3$ by the surface $N(S)$ of its normal lines in the space $\mathrm{L}(\mathbb{R}^3)$ of all affine lines of $\mathbb{R}^3$.
2. **Kaehler metric** This space is four dimensional and admits a notion of angular momentum, which allows one to measure distances between such lines. The umbilic of S then appears as a complex point of $N(S)$.
3. **Mean curvature flow** We deform $N(S) \subset \mathrm{L}(\mathbb{R}^3)$ into a surface of largest twist, namely maximal area, which is seen to be holomorphic.
4. **Baking the cake** This holomorphic surface now allows us to separate $N(S)$ locally from itself by a Hamiltonian isotopy, which results in an upper bound on the index of $N(S)$ and thereby an upper bound on the umbilic index of S, the bound being one.