DIRICHLET SERIES CONSTRUCTED FROM PERIODS OF AUTOMORPHIC FORMS

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We consider certain Dirichlet series constructed from periods of automorphic forms. Let k be a fixed natural number, Γ be a co-finite torsion-free discrete subgroup of $PSL(2,\mathbb{R})$. For $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in PSL(2,\mathbb{R})$, put $Q_\gamma(z) = cz^2 + (d-a)z - b$.

Definition 1 (Periods of automorphic forms)**.** *Let g be a weight $4k$ holomorphic cusp form for Γ and γ be a hyperbolic element in Γ. The period integral of g over the closed geodesic associated to γ is defined by*

$$\alpha_{2k}(\gamma, g) = \int_{z_0}^{\gamma z_0} Q_\gamma(z)^{2k-1} g(z)\, dz. \tag{1}$$

This integral does not depend on the choice on the point $z_0 \in H$ and the path from z_0 and γz_0.

It is known that these periods are important to arithmetic theory of automorphic forms.

Let $\mathrm{Prim}(\Gamma)$ be the set of primitive hyperbolic conjugacy classes of Γ. For a hyperbolic element $\gamma \in \Gamma$, put $\ell(\gamma)$ be the length of the geodesic associated to γ and $N(\gamma) = \exp(\ell(\gamma))$.

Definition 2 (Dirichlet series $\Xi_\Gamma(s; g)$)**.** *For $g \in S_{4k}(\Gamma)$ and $s \in \mathbb{C}$ with $\mathrm{Re}\, s > 1$, define*

$$\begin{aligned} \Xi_\Gamma(s; g) &= \sum_{\gamma \in \mathrm{Prim}(\Gamma)} \sum_{m=1}^{\infty} \beta_{2k}(\gamma, g)\, N(\gamma)^{-ms} \\ &= \sum_{\gamma \in \mathrm{Prim}(\Gamma)} \beta_{2k}(\gamma, g)\, \frac{N(\gamma)^{-s}}{1 - N(\gamma)^{-s}} \end{aligned} \tag{2}$$

with

$$\beta_{2k}(\gamma, g) = \frac{\overline{\alpha_{2k}(\gamma, g)}}{2^{6k-3} \sinh^{2k-1}(\ell(\gamma)/2)}. \tag{3}$$

This series is absolutely convergent for $\mathrm{Re}\, s > 1$.

Our concern is to investigate analytic properties of $\Xi_\Gamma(s; g)$.

Theorem 3. *Let Γ be a co-compact torsion-free discrete subgroup of $PSL(2,\mathbb{R})$ and $g \in S_{4k}(\Gamma)$. The function $\Xi_\Gamma(s; g)$, defined for $\mathrm{Re}\, s > 1$, has the analytic continuation as a meromorphic function on the whole complex plane. $\Xi_\Gamma(s; g)$ has simple poles located at $s = \frac{1}{2} - l \pm ir_n$, $(l \geq 0, n \geq 1)$. There are no poles other than described as above. Here $\{1/4 + r_n^2\}$ are eigenvalues of the Laplacian acting on $L^2(\Gamma \backslash H)$. $\Xi_\Gamma(s; g)$ satisfy the functional equation*

$$\Xi_\Gamma(-s; g) = \Xi_\Gamma(s; g). \tag{4}$$

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