Title: "Similarity transformations, Quantization and Classical Mechanics"

Lecturer: ATLOM=A Tiny Little Old Mathematician

Date: 25th October 2007, pm 3.00-4.30

Abstract: Fix a Hilbert space $\mathfrak{H}$. Prepairing a self-adjoint operator $\mathbf{H}$, we transform a given operator $\mathbf{P}$ using the unitary operator $\mathbf{U}(t) = e^{i\hbar^{-1}t\mathbf{H}}$ (here, mathematically $\hbar > 0$ is an auxiliary parameter) as

$$\mathbf{P}(t) = \mathbf{U}(t)^*\mathbf{P}\mathbf{U}(t).$$

Formal calculation yields

$$\dot{\mathbf{P}}(t) = i\hbar^{-1}[\mathbf{H}, \mathbf{P}(t)] \quad \text{with} \quad \mathbf{P}(0) = \mathbf{P}. \tag{1}$$

This formula (1) is called the Heisenberg picture in Quantum Mechanics, therefore in general there exists a Classical Mechanics corresponding to (1). This point of view is embodied in Egorov's Theorem of the linear PDE theory.

If $\mathfrak{H}$ equals to $\mathbb{C}^n$, the similarity transformations in linear algebra are well-known. Therefore, a primitive question is whether there exists a classical mechanics to this finite dimensional case?

My solution is affirmative, there exists CM!
Main ideas are
(i) at least $2^m \times 2^m$-matrix is decomposed by matrices satisfying Clifford relations,
(ii) Cliford albgebra has a differential representation in Grassmann algebra, and
(iii) we may develop analysis based on Grassmann algebra which is called superanalysis.

Applying this idea, we may give a generaization of Egorov's Theorem for a system of PDEs.

Important remark: We need a countable number of Grassmann generators to form "even and odd" or "bosonic and ferminioc" variables, which are two of *New dimensions in geometry* by Y.I. Manin. If I remembered right, he mentioned except above two, "arithmetic dimension" which I have never touched in my short life in mathematics.